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ABSTRACT: We consider $\tan\beta$-enhanced quantum effects in the minimal supersymmetric standard model (MSSM) including those from the Higgs sector. To this end, we match the MSSM to an effective two-Higgs doublet model (2HDM), assuming that all SUSY particles are heavy, and calculate the coefficients of the operators that vanish or are suppressed in the MSSM at tree-level. Our result clarifies the dependence of the large-$\tan\beta$ resummation on the renormalization convention for $\tan\beta$, and provides analytic expressions for the Yukawa and trilinear Higgs interactions. The numerical effect is analyzed by means of a parameter scan, and we find that the Higgs-sector effects, where present, are typically larger than those from the "wrong-Higgs" Yukawa couplings in the 2HDM.

Contents

1. Introduction

It is well-known that loop effects in the minimal supersymmetric extension of the standard model (MSSM) can become large when the ratio $\tan\beta = v_u/v_d$ of the two Higgs vacuum expectation values is sizeable. An important example is the Higgs coupling to bottom quarks, $Hb\bar{b}$ [1, 2]. In the limit of heavy superpartner particle masses, the relevant couplings are [3]

$$y_D H_d \epsilon Q D - \delta\tilde{y}_D H_u^C \epsilon Q D + \text{h.c.}$$
$$\to -(y_b v_d + \delta\tilde{y}_b v_u)\, \bar{b}_R b_L - y_b H_d^0 \bar{b}_R b_L - \delta\tilde{y}_b H_u^{0*} \bar{b}_R b_L + \text{h.c.} \tag{1.1}$$

This includes a coupling $\delta\tilde{y}_D$ to the "wrong" Higgs doublet, which is forbidden by supersymmetry at tree level. The quantity $\delta\tilde{y}_D$ is an ordinary one-loop effect and not enhanced in any way. However, due to the large ratio of vacuum expectation values (vevs) the bottom quark mass

$$m_b = y_b v_d + \delta\tilde{y}_b v_u = m_b^{(0)} \left(1 + \frac{\delta\tilde{y}_b}{y_b}\tan\beta\right) \tag{1.2}$$

receives large $\tan\beta$-enhanced corrections to its tree-level value $m_b^{(0)} = y_b v_d$ [4]. So does the $Hb\bar{b}$ coupling, when y_b is eliminated in favour of m_b as is usually done. In general, while the

loop effect is not large by itself, it provides large corrections to couplings or observables, which are suppressed by the small vacuum expectation value v_d or which vanish at tree level. This type of relative enhancement is clearly a one-loop effect, which does not repeat itself in higher loop orders.

Here we investigate other effects of this type from the Higgs sector, where loop effects lead to a mixing of the H_u and H_d^* fields through two- and four-point interactions, which indirectly modify the Yukawa couplings to the physical Higgs bosons. We also address the issue of renormalization conventions for $\tan\beta$ and their stability against radiative corrections, as investigated in a numerical approach in [5, 6]. The indirect effects from the Higgs sector have not been considered explicitly in the literature, although our results are implicit in one-loop calculations of MSSM Higgs decay rates [1, 2]. However, these calculations are usually available only in numerical form. Having simple analytic expressions at hand is often useful to acquire a better understanding of parameter dependences, especially for the MSSM with its large parameter space. Higgs-sector effects from two-point interactions have been considered recently in [7], while their consequences for $B\bar{B}$ mixing are discussed in [8].

The theoretical issues can be exposed most clearly in the decoupling limit, assuming that all superpartner particles are heavy, of order M_{SUSY}, while the standard model (SM) particles together with all five physical Higgs bosons are light, at most $O(M_{\text{EW}})$, the electroweak scale. In practice, the decoupling limit may be a good approximation for M_{SUSY} as small as a few hundred GeV. The assumed hierarchy implies a certain amount of fine-tuning, since the soft SUSY breaking parameters and the μ parameter must be $O(M_{\text{SUSY}})$, while $\hat{m}_{u,d}^2 \equiv |\mu|^2 + m_{u,d}^2$ are $O(M_{\text{EW}})$ to make the Higgs bosons light. The Higgs-mixing parameter b must even be much smaller than M_{EW}^2 to achieve large $\tan\beta$. The first fine-tuning is the well-known "little hierarchy problem", here built in by hand by the assumption $M_{\text{SUSY}} \gg M_{\text{EW}}$, but the second is an extra fine-tuning of b specific to the large-$\tan\beta$ scenario, which has received less attention up to now. To simplify the discussion we consider only the third family of fermions and assume CP conservation in the MSSM. Including the CKM matrix and CP-violating MSSM phases would be straightforward. Under these assumptions we match the MSSM to an effective general two-Higgs doublet model (2HDM), and determine the physical Higgs fields, the Yukawa couplings, and trilinear Higgs couplings from the one-loop corrected effective 2HDM.

2. Matching the MSSM to the 2HDM

The Higgs and Yukawa terms of the MSSM Lagrangian are given by

$$
\begin{aligned}
\mathcal{L} = {} & (D_\mu H_d)^\dagger (D^\mu H_d) + (D_\mu H_u)^\dagger (D^\mu H_u) - \hat{m}_d^2 H_d^\dagger H_d - \hat{m}_u^2 H_u^\dagger H_u \\
& - b\left(H_d \epsilon H_u - H_u^\dagger \epsilon H_d^\dagger\right) - \frac{g_2^2}{2}\left(H_d^\dagger H_u\right)\left(H_u^\dagger H_d\right) - \frac{1}{8}(g_2^2 + g_1^2)\left(H_d^\dagger H_d - H_u^\dagger H_u\right)^2 \\
& - y_t H_u \epsilon Q U + y_b H_d \epsilon Q D + y_\tau H_d \epsilon L E + \text{h.c.}
\end{aligned} \tag{2.1}
$$

The conventions are as follows: the antisymmetric 2×2 matrix ϵ is defined with $\epsilon_{12} = -1$. The hypercharge U(1)$_Y$ and SU(2) couplings are g_1 and g_2, respectively. The quark fields are $Q_\alpha = (t_{L\alpha}, b_{L\alpha})^T$, $U^\alpha = (t_R^*)^\alpha$, $D^\alpha = (b_R^*)^\alpha$, the leptons $L_\alpha = (\nu_{L\alpha}, \tau_{L\alpha})^T$, $E^\alpha = (\tau_R^*)^\alpha$,

where $\alpha = 1, 2$ denotes a Weyl spinor index. For the Higgs doublets we also use the notation $\Phi_1 = H_d$, $\Phi_2 = H_u^C = \epsilon H_u^*$, such that Φ_1 and Φ_2 both have hypercharge $-1/2$. Our MSSM conventions follow [9] except for $y_b = -[y_b]_{\text{Rosiek}}$, $y_\tau = -[y_\tau]_{\text{Rosiek}}$, $b = -[m_{12}^2]_{\text{Rosiek}}$ and $A_t = -[A_t]_{\text{Rosiek}}$. In this way our definitions of the Yukawa couplings and A parameters are consistent with the SPA convention [10].

2.1 Parameters and renormalization of the MSSM

The relevant parameters of the MSSM Higgs sector are g_1, g_2, $\hat{m}_{u,d}^2$, b, $y_{b,t}$ and the two vevs $v_{u,d}$. It appears to be contrary to the spirit of interpreting the MSSM as the high-scale theory and the 2HDM as the low-energy effective theory to introduce the vevs as MSSM parameters, since the vevs should be found *a posteriori* by minimizing the 2HDM effective potential. However, since we wish to use standard MSSM renormalization conventions, and moreover introduce $\tan\beta$ as one of the MSSM parameters, we substitute

$$H_d \to \begin{pmatrix} v_d + H_d^0 \\ H_d^- \end{pmatrix}, \qquad H_u \to \begin{pmatrix} H_u^+ \\ v_u + H_u^0 \end{pmatrix} \tag{2.2}$$

in the Lagrangian (2.1). In this way, after further fixing the renormalization convention, the parameter $\tan\beta = v_u/v_d$ is well-defined, contrary to the general 2HDM, where $\tan\beta$ is unphysical, since it can assume any value by performing a rotation in the (Φ_1, Φ_2) fields. In the MSSM such rotations are excluded by holomorphy of the superpotential.

An equivalent set of parameters consists of

$$e, M_W, M_Z, M_A, \tan\beta, t_{u,d}, m_{b,t}, \tag{2.3}$$

related to the previous set by

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}},$$

$$M_W = \frac{g_2}{\sqrt{2}} v, \qquad\qquad M_Z = \frac{1}{\sqrt{2}} \sqrt{g_1^2 + g_2^2}\, v,$$

$$M_A^2 = \hat{m}_u^2 + \hat{m}_d^2, \qquad\qquad \tan\beta = \frac{v_u}{v_d},$$

$$t_d = \hat{m}_d^2 v_d + \frac{1}{4}\left(g_1^2 + g_2^2\right) v_d \left(v_d^2 - v_u^2\right) - b v_u,$$

$$t_u = \hat{m}_u^2 v_u + \frac{1}{4}\left(g_1^2 + g_2^2\right) v_u \left(v_u^2 - v_d^2\right) - b v_d, \tag{2.4}$$

with $v = \sqrt{v_u^2 + v_d^2}$. The quark masses are given by $m_{b,t} = y_{b,t} v_{d,u}$ at tree level. For vanishing tadpole couplings $t_{u,d}$, M_A is the mass of the CP-odd neutral Higgs boson.

The parameters so far are bare parameters. We introduce renormalized parameters by substituting $p \to p_0 \to p + \Delta p$, where now p_0 (p) is the bare (renormalized) parameter and Δp the corresponding counterterm. The renormalization conventions are specified by imposing conditions on the parameters of the second set: e, M_W, M_Z, M_A are defined by the conventional on-shell renormalization conditions for the electric charge and the

particle pole masses, the tadpole couplings $t_{u,d}$ by the condition that the tadpoles (one-point functions) vanish. The renormalization condition for $\tan\beta$ and the Yukawa couplings, and the definition of $m_{b,t}$ beyond tree level will be discussed later. The counterterms Δp for the parameters of the first set are then defined by requiring that (2.4) holds for the bare as well as the renormalized parameters. Specifically, for $\tan\beta$ this implies

$$\tan\beta + \Delta t_\beta = [\tan\beta]_0 = \frac{[v_u]_0}{[v_d]_0} = \frac{v_u + \Delta v_u}{v_d + \Delta v_d} = \frac{v_u}{v_d} + \frac{v_u}{v_d}\left(\frac{\Delta v_u}{v_u} - \frac{\Delta v_d}{v_d}\right) + \cdots . \quad (2.5)$$

Employing $\tan\beta = v_u/v_d$, the one-loop relation

$$\Delta t_\beta = \tan\beta\left(\frac{\Delta v_u}{v_u} - \frac{\Delta v_d}{v_d}\right) \quad (2.6)$$

follows. Similarly, one obtains a set of equations relating the shifts of all parameters of the second set to those of the first.

We do not need to discuss the renormalization of superpartner masses and the A and μ terms, since these occur only in loops. In a one-loop calculation, it is sufficient to know these parameters at tree-level. Other SM parameters like the strong coupling or CKM matrix play no role here. For the subsequent discussion of $\tan\beta$-enhanced terms, field renormalization in the MSSM can also be ignored. To see this, introduce the renormalization factors Z_u, Z_d for the SU(2)-doublet fields according to $[H_u]_0 = Z_u^{1/2}H_u$ and $[H_d]_0 = Z_d^{1/2}H_d$. The bare vevs are now written as $[v_i]_0 = Z_i^{1/2}v_i + \widetilde{\Delta}v_i$, $i = u,d$. The vev counterterms $\widetilde{\Delta}v_i$ are finite quantities, since the theory in the broken phase can be rendered finite by the same counterterms as the unbroken theory before the shift (2.2). The one-loop relation between the vev counterterms Δv_i defined previously and the vev counterterms $\widetilde{\Delta}v_i$ is simply given by $\Delta v_i = v_i\delta Z_i/2 + \widetilde{\Delta}v_i$. Since the field renormalization factors Z_i are not enhanced in the large-$\tan\beta$ limit, we have $\Delta v_d \approx \widetilde{\Delta}v_d$. The $\tan\beta$-counterterm including field renormalization reads $\widetilde{\Delta}t_\beta = \Delta t_\beta - (\delta Z_u - \delta Z_d)/2$. In the large-$\tan\beta$ limit it follows from (2.6) that $\Delta t_\beta \approx -\tan\beta\,\Delta v_d/v_d \approx -\tan\beta\,\widetilde{\Delta}v_d/v_d$. Thus the field renormalization factors drop out in the large-$\tan\beta$ approximation, and the counterterm Δt_β is a finite quantity, whose value is nevertheless fixed by the imposed renormalization conventions.

2.2 The effective 2HDM Lagrangian

We integrate out the heavy SUSY particles and expand the MSSM effective action in local operators. The result is a general 2HDM, where the Higgs-Yukawa terms read

$$\begin{aligned}
\mathcal{L}_{2\text{HDM}} = {}& Z_{ab}(D_\mu\Phi_a)^\dagger(D^\mu\Phi_b) - m_{ab}^2\Phi_a^\dagger\Phi_b \\
& - \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 - \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 - \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
& - \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \left(\lambda_6\Phi_1^\dagger\Phi_1 + \lambda_7\Phi_2^\dagger\Phi_2\right)\Phi_1^\dagger\Phi_2 + \text{h.c.}\right] \\
& + Y_b\Phi_1\epsilon QD + Y_t\Phi_2^\dagger QU + Y_\tau\Phi_1\epsilon LE - \delta\tilde{y}_b\Phi_2\epsilon QD + \delta\tilde{y}_t\Phi_1^\dagger QU - \delta\tilde{y}_\tau\Phi_2\epsilon LE + \text{h.c.}
\end{aligned} \quad (2.7)$$

neglecting operators with dimension higher than four. The last line equals

$$Y_bH_d\epsilon QD - Y_tH_u\epsilon QU + Y_\tau H_d\epsilon LE - \delta\tilde{y}_bH_u^C\epsilon QD + \delta\tilde{y}_tH_d^C\epsilon QU - \delta\tilde{y}_\tau H_u^C\epsilon LE + \text{h.c.}$$
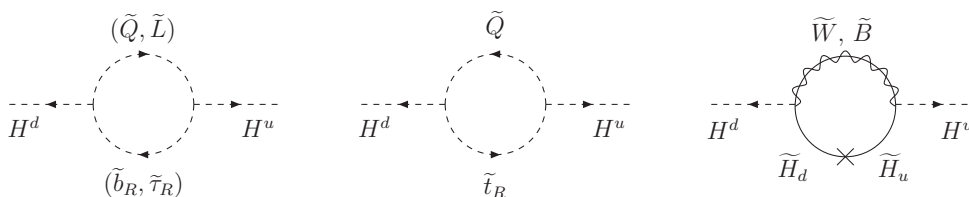
Figure 1: Diagrams entering the matching of m_{12}^2 and z_{12}. The fields $\widetilde{B}$ and $\widetilde{W}$ are the spin-1/2 superpartners of the electroweak gauge bosons. For internal lines, the notation X, Y means that there are contributions from both particles, while the parentheses (X_i, Y_i) in the first diagram indicate that there is a X_i (squark) and a Y_i (slepton) species loop contribution.

in terms of the $H_{u,d}$ fields, so the Yukawa couplings take their standard form. Since we assumed CP conservation in the MSSM, the matrices Z_{ab}, m_{ab}^2 $(a, b = 1, 2)$ and the couplings λ_i are real. Note that the kinetic terms of the Higgs fields are not yet canonical, because we match 1PI Green functions. The kinetic terms of the fermions can be disregarded, since their renormalization is an ordinary one-loop correction to an unsuppressed tree term. At tree-level

$$
\begin{aligned}
Z_{ab} &= \delta_{ab}, \\
m_{11}^2 &= \hat{m}_d^2, & m_{22}^2 &= \hat{m}_u^2, & m_{12}^2 = m_{21}^2 &= b, \\
\lambda_1 = \lambda_2 &= \frac{1}{4}(g_1^2 + g_2^2), & \lambda_3 &= \frac{1}{4}(g_2^2 - g_1^2), & \lambda_4 &= -\frac{1}{2}g_2^2, \\
\lambda_5 = \lambda_6 = \lambda_7 &= 0, \\
Y_{b,t,\tau} &= y_{b,t,\tau}, & \delta\tilde{y}_b = \delta\tilde{y}_t = \delta\tilde{y}_\tau &= 0.
\end{aligned}
\tag{2.8}
$$

In the following the one-loop corrections to the couplings that vanish or are small at tree-level are of particular interest: $\delta Z_{12} \equiv z_{12}$, δm_{12}^2, $\delta\lambda_{5,6,7}$, and $\delta\tilde{y}_{b,t,\tau}$. The result of the calculation is summarized in the next section.

2.3 One-loop results

All matching calculations are performed in the $SU(2) \times U(1)_Y$ symmetric "phase" of the MSSM. The couplings z_{12} and m_{12}^2 follow from an expansion of the off-diagonal $H_u H_d$ two-point function in its external momentum. The relevant one-loop diagrams with heavy superpartner particles in the loop are shown in figure 1. For the kinetic-mixing term we find

$$
\begin{aligned}
z_{12} = \mu^* \Big\{ &3y_t A_t^* H_2\big(m_{\tilde{t}_R}, m_{\tilde{Q}}\big) + 3y_b A_b^* H_2\big(m_{\tilde{b}_R}, m_{\tilde{Q}}\big) + y_\tau A_\tau^* H_2\big(m_{\tilde{\tau}_R}, m_{\tilde{L}}\big) \\
&+ 3g_2^2 M_2 H_2(|\mu|, M_2) + g_1^2 M_1 H_2(|\mu|, M_1) \Big\}
\end{aligned}
\tag{2.9}
$$

For the off-diagonal mass term, we write $m_{12}^2 = b + \delta b + [\Delta b]_{\text{heavy}}$ and obtain for δb the

expression (2.9) with the replacement $H_2(m_1, m_2) \to -I_2(m_1, m_2)$, where

$$I_2(m_1, m_2) = \frac{1}{16\pi^2} \left(\frac{1}{\epsilon} - \ln \frac{m_1^2}{\nu^2} + i_2(x) \right) ,$$

$$i_2(x) = 1 + \frac{x \ln x}{1 - x} , \tag{2.10}$$

$$H_2(m_1, m_2) = \frac{1}{16\pi^2} \frac{1}{2m_1^2} h_2(x) ,$$

$$h_2(x) = \frac{1 - x^2 + 2x \ln x}{(1 - x)^3} , \tag{2.11}$$

and $x \equiv m_2^2/m_1^2$. The kinetic-mixing correction z_{12} is finite, as it should be, while the correction to m_{12}^2 is ultraviolet divergent and requires regularization (here dimensional regularization, space-time dimension $d = 4 - 2\epsilon$ and renormalization scale ν). The $1/\epsilon$ pole is cancelled by the counterterm contribution $[\Delta b]_{\text{heavy}}$ to m_{12}^2. The counterterm contribution is the difference between the MSSM and the 2HDM counterterm, which survives the matching relation, and cancels the singularity in the heavy-particle loops. In the following we shall drop the label "heavy" in the counterterm contributions. We remark that we have kept complex conjugation on the μ and A parameters, where required, if they were complex. However, by our assumption of CP-conservation, μ and $A_{t,b,\tau}$ are real. Here and below the masses that enter the arguments of the loop functions are the gaugino and sfermion mass parameters in the soft supersymmetry-breaking part of the MSSM Lagrangian and not the physical mass parameters. The physical masses receive additional contributions at tree level proportional to the vevs $v_{u,d}$ from electroweak symmetry breaking, which are of higher order in the $M_{\text{EW}}/M_{\text{SUSY}}$ expansion.

For the four-point Higgs couplings $\lambda_{5,6,7}$, which vanish at tree level, we obtain the ultraviolet-finite one-loop expressions

$$
\begin{aligned}
\delta\lambda_5 = {}& -3(\mu^* A_t^*)^2 y_t^2 K_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) - 3(\mu^* A_b^*)^2 y_b^2 K_2(m_{\tilde{Q}}, m_{\tilde{b}_R}) \\
& - (\mu^* A_\tau^*)^2 y_\tau^2 K_2(m_{\tilde{L}}, m_{\tilde{\tau}_R}) \\
& + (\mu^*)^2 M_1^2 g_1^4 K_2(|\mu|, M_1) + 3(\mu^*)^2 M_2^2 g_2^4 K_2(|\mu|, M_2) \\
& + 2(\mu^*)^2 M_1 M_2 g_1^2 g_2^2 K_3(|\mu|, M_1, M_2) , \\
\delta\lambda_6 = {}& -3\mu^* A_b^* y_b^3 \left\{ J_2(m_{\tilde{Q}}, m_{\tilde{b}_R}) + J_2(m_{\tilde{b}_R}, m_{\tilde{Q}}) \right\} \\
& - \mu^* A_\tau^* y_\tau^3 \left\{ J_2(m_{\tilde{L}}, m_{\tilde{\tau}_R}) + J_2(m_{\tilde{\tau}_R}, m_{\tilde{L}}) \right\} \\
& + 3\mu^* y_t A_t^* \left(-\frac{1}{4} \right) \left\{ \left[g_2^2 - \frac{g_1^2}{3} \right] J_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) + \left[\frac{4g_1^2}{3} \right] J_2(m_{\tilde{t}_R}, m_{\tilde{Q}}) \right\} \\
& + 3\mu^* y_b A_b^* \left(-\frac{1}{4} \right) \left\{ \left[-g_2^2 - \frac{g_1^2}{3} \right] J_2(m_{\tilde{Q}}, m_{\tilde{b}_R}) + \left[-\frac{2g_1^2}{3} \right] J_2(m_{\tilde{b}_R}, m_{\tilde{Q}}) \right\} \\
& + \mu^* y_\tau A_\tau^* \left(-\frac{1}{4} \right) \left\{ \left[-g_2^2 + g_1^2 \right] J_2(m_{\tilde{L}}, m_{\tilde{\tau}_R}) + \left[-2g_1^2 \right] J_2(m_{\tilde{\tau}_R}, m_{\tilde{L}}) \right\} \\
& - 3\mu^* |\mu|^2 A_t^* y_t^3 K_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) - 3\mu^* |A_b|^2 A_b^* y_b K_2(m_{\tilde{Q}}, m_{\tilde{b}_R}) \\
& - \mu^* |A_\tau|^2 A_\tau^* y_\tau K_2(m_{\tilde{L}}, m_{\tilde{\tau}_R})
\end{aligned}
\tag{2.12}
$$

$$-3\mu^* M_2 g_2^4 L_2(|\mu|, M_2) - \mu^* M_1 g_1^4 L_2(|\mu|, M_1)$$
$$-\mu^* (M_2 + M_1) g_2^2 g_1^2 L_3(|\mu|, M_2, M_1),\tag{2.13}$$
$$\delta\lambda_7 = -3\mu^* A_t^* y_t^3 \left\{ J_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) + J_2(m_{\tilde{t}_R}, m_{\tilde{Q}}) \right\}$$
$$-3\mu^* y_t A_t^* \left(-\frac{1}{4}\right) \left\{ \left[g_2^2 - \frac{g_1^2}{3}\right] J_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) + \left[\frac{4g_1^2}{3}\right] J_2(m_{\tilde{t}_R}, m_{\tilde{Q}}) \right\}$$
$$-3\mu^* y_b A_b^* \left(-\frac{1}{4}\right) \left\{ \left[-g_2^2 - \frac{g_1^2}{3}\right] J_2(m_{\tilde{Q}}, m_{\tilde{b}_R}) + \left[-\frac{2g_1^2}{3}\right] J_2(m_{\tilde{b}_R}, m_{\tilde{Q}}) \right\}$$
$$-\mu^* y_\tau A_\tau^* \left(-\frac{1}{4}\right) \left\{ \left[-g_2^2 + g_1^2\right] J_2(m_{\tilde{L}}, m_{\tilde{\tau}_R}) + \left[2g_1^2\right] J_2(m_{\tilde{\tau}_R}, m_{\tilde{L}}) \right\}$$
$$-3\mu^* |A_t|^2 A_t^* y_t K_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) - 3\mu^* |y_b \mu^*|^2 A_b^* y_b K_2(m_{\tilde{Q}}, m_{\tilde{b}_R})$$
$$-\mu^* |y_\tau \mu^*|^2 A_\tau^* y_\tau K_2(m_{\tilde{L}}, m_{\tilde{\tau}_R})$$
$$-3\mu^* M_2 g_2^4 L_2(|\mu|, M_2) - \mu^* M_1 g_1^4 L_2(|\mu|, M_1)$$
$$-\mu^* (M_2 + M_1) g_2^2 g_1^2 L_3(|\mu|, M_2, M_1).\tag{2.14}$$

The diagrams relevant for the computation of $\delta\lambda_7$ are shown in figure 2. The loop functions read

$$J_2(m_1, m_2) = \frac{1}{16\pi^2} \left(-\frac{1}{m_1^2}\right) j_2(x),$$
$$j_2(x) = \frac{1}{1-x} + \frac{x \ln x}{(1-x)^2},\tag{2.15}$$
$$K_2(m_1, m_2) = \frac{1}{16\pi^2} \left(-\frac{1}{m_1^4}\right) k_2(x),$$
$$k_2(x) = \frac{2}{(1-x)^2} + \frac{(1+x) \ln x}{(1-x)^3},\tag{2.16}$$
$$K_3(m_1, m_2, m_3) = \frac{1}{16\pi^2} \left(-\frac{1}{m_1^4}\right) k_3(x, y),$$
$$k_3(x, y) = \frac{1}{(1-x)(1-y)} + \frac{x \ln x}{(x-y)(1-x)^2} + \frac{y \ln y}{(y-x)(1-y)^2},$$
$$L_2(m_1, m_2) = \frac{1}{16\pi^2} \left(-\frac{1}{m_1^2}\right) l_2(x),$$
$$l_2(x) = \frac{1+x}{(1-x)^2} + \frac{2x \ln x}{(1-x)^3},\tag{2.17}$$
$$L_3(m_1, m_2, m_3) = \frac{1}{16\pi^2} \left(-\frac{1}{m_1^2}\right) l_3(x, y),$$
$$l_3(x, y) = \frac{1}{(1-x)(1-y)} + \frac{x^2 \ln x}{(x-y)(1-x)^2} + \frac{y^2 \ln y}{(y-x)(1-y)^2},\tag{2.18}$$

with $x \equiv m_2^2/m_1^2$, $y \equiv m_3^2/m_1^2$.

The loop-induced Yukawa couplings to the "wrong" Higgs field are given by

$$\delta\tilde{y}_b = -\frac{8}{3}\mu^* y_b M_3 g_3^2 J_3(M_3, m_{\tilde{Q}}, m_{\tilde{b}_R}) - \mu^* y_b y_t A_t^* J_3(|\mu|, m_{\tilde{Q}}, m_{\tilde{t}_R})$$
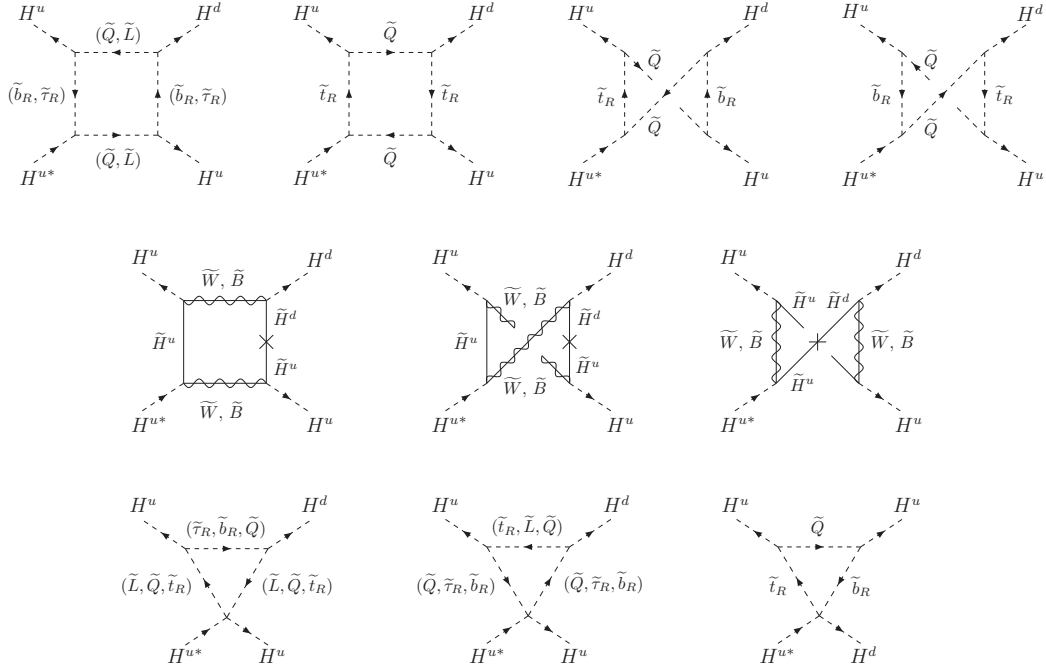
Figure 2: Diagrams entering the matching of $\delta\lambda_7$.

$$+ \frac{3}{2}\mu^* y_b M_2\, g_2^2 J_3(|\mu|, M_2, m_{\tilde{Q}}) + \frac{1}{9}\mu^* y_b M_1\, g_1^2 J_3(M_1, m_{\tilde{Q}}, m_{\tilde{b}_R})$$

$$+ \frac{1}{3}\mu^* y_b M_1\, g_1^2 J_3(|\mu|, M_1, m_{\tilde{b}_R}) + \frac{1}{6}\mu^* y_b M_1\, g_1^2 J_3(|\mu|, M_1, m_{\tilde{Q}})\,, \qquad (2.19)$$

$$\delta\tilde{y}_t = \frac{8}{3}\mu^* y_t M_3\, g_3^2 J_3(M_3, m_{\tilde{Q}}, m_{\tilde{t}_R}) + \mu^* y_b y_t A_b^* J_3(|\mu|, m_{\tilde{Q}}, m_{\tilde{b}_R})$$

$$- \frac{3}{2}\mu^* y_t M_2\, g_2^2 J_3(|\mu|, M_2, m_{\tilde{Q}}) + \frac{2}{9}\mu^* y_t M_1\, g_1^2 J_3(M_1, m_{\tilde{Q}}, m_{\tilde{t}_R})$$

$$- \frac{2}{3}\mu^* y_t M_1\, g_1^2 J_3(|\mu|, M_1, m_{\tilde{t}_R}) + \frac{1}{6}\mu^* y_t M_1\, g_1^2 J_3(|\mu|, M_1, m_{\tilde{Q}})\,, \qquad (2.20)$$

$$\delta\tilde{y}_\tau = \frac{3}{2}\mu^* y_\tau M_2\, g_2^2 J_3(|\mu|, M_2, m_{\tilde{L}}) - \mu^* y_\tau M_1\, g_1^2 J_3(M_1, m_{\tilde{L}}, m_{\tilde{\tau}_R})$$

$$+ \mu^* y_\tau M_1\, g_1^2 J_3(|\mu|, M_1, m_{\tilde{\tau}_R}) - \frac{1}{2}\mu^* y_\tau M_1\, g_1^2 J_3(|\mu|, M_1, m_{\tilde{L}})\,, \qquad (2.21)$$

where

$$J_3(m_1, m_2, m_3) = \frac{1}{16\pi^2}\left(\frac{1}{m_1^2}\right) j_3(x, y)\,,$$

$$j_3(x, y) = \frac{x \ln x}{(x - y)(1 - x)} + \frac{y \ln y}{(y - x)(1 - y)}\,. \qquad (2.22)$$

The diagrams that enter the computation of $\delta\tilde{y}_b$ are shown in figure 3. The expression for $\delta\tilde{y}_b$ agrees with the well-known result for the $\tan\beta$-enhanced correction to the bottom quark mass [3, 4], when the bino contribution (proportional to M_1) and terms of order
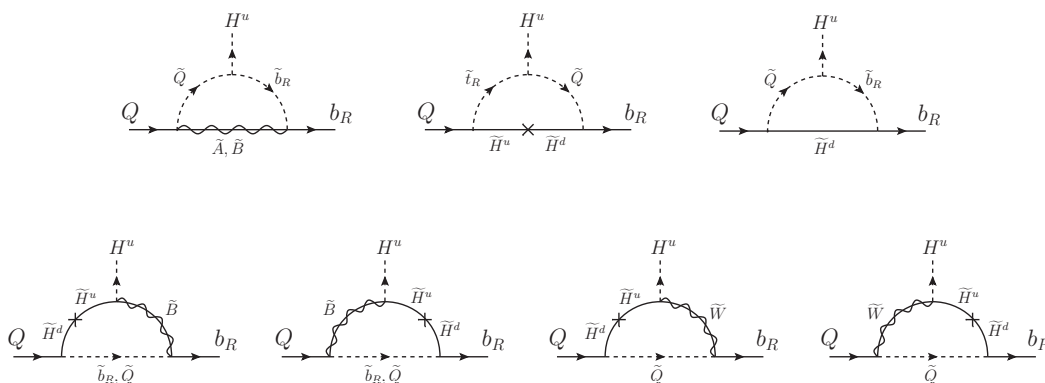
Figure 3: Diagrams entering the matching of $\delta\tilde{y}_b$. The gluino is denoted by $\widetilde{A}$.

$M_{\rm EW}/M_{\rm SUSY}$ are neglected. Our full results for $\delta\tilde{y}_b/y_b$ and $\delta\tilde{y}_t/y_t$ agree with the expressions for the correspondent quantities $\epsilon_0 + \epsilon_Y y_t^2$ and $-[\epsilon_0' + \epsilon_Y' y_b^2]$ given in [7], which were written in terms of the rescaled A-parameters, $A_{b,t} \to y_{b,t}A_{b,t}$. The one-loop corrections $\delta\lambda_{5-7}$ agree with those given in [20], which assumed all squark masses equal to $M_{\rm SUSY}$ and neglected the gaugino and slepton contributions. Our result for $\delta\lambda_5$ agrees with [8], where the bino contributions have been neglected.

It is no surprise that the one-loop effects of interest are all proportional to the μ-parameter, since the off-diagonal two-point terms and the $\lambda_{5,6,7}$ couplings are protected by a $U(1)_{\rm PQ}$ symmetry that is broken in the MSSM only by the μ and b term [4]. Note that the quantum correction to m_{12}^2 is of $O(M_{\rm SUSY}^2)$, while the b (or rather m_{12}^2) parameter was assumed to be of order $M_{\rm EW}^2/\tan\beta$. Thus, the large-$\tan\beta$ scenario requires more severe fine-tuning (by a factor of $\tan\beta$) than the one to maintain the "little hierarchy" $M_{\rm EW} \ll M_{\rm SUSY}$. It therefore appears less natural in this respect than the generic MSSM.

3. Higgs couplings

We are now ready to derive the one-loop corrected Higgs couplings, which are $\tan\beta$-enhanced relative to their tree-level expressions. For any quantity $\mathcal{G}$ we can write its NLO MSSM expression as

$$\mathcal{G} = \mathcal{G}_{\rm tree} + \mathcal{G}_{\rm heavy} + \mathcal{G}_{\rm light}, \tag{3.1}$$

where $\mathcal{G}_{\rm heavy}$ is the contribution from the scale $M_{\rm SUSY}$, in practice from loops containing SUSY particles including the corresponding counterterms, and $\mathcal{G}_{\rm light}$ the remaining contribution from the electroweak scale (standard model particles and Higgses).

We will be interested only in effective interactions that vanish or are suppressed at tree-level by a factor of $\tan\beta$ relative to their natural size, and we shall compute them at one loop only at leading order in $1/\tan\beta$. In this case $\mathcal{G}_{\rm light}$ is suppressed relative to $\mathcal{G}_{\rm heavy}$ and can be set to zero. This can be understood from the fact that these interactions are protected by the PQ-symmetry mentioned above. This symmetry is broken strongly by the μ parameter that enters the high-scale contributions $\mathcal{G}_{\rm heavy}$, but only weakly in the

low-energy contributions by the small b (or rather m_{12}^2, but in one-loop diagrams there is no distinction) parameter, $b \sim M_A^2/\tan\beta$. The low-energy contributions $\mathcal{G}_{\text{light}}$ all involve an insertion of b, or an explicit coupling to the small vev $v_d \sim M_{\text{EW}}/\tan\beta$, and are therefore negligible. On the other hand, the contribution $\mathcal{G}_{\text{heavy}}$ from the superpartner particle loops is already encoded in the effective couplings of the 2HDM (2.7). We therefore conclude that the leading one-loop contributions to the tree-suppressed Higgs couplings in the MSSM are simply obtained by computing the physical Higgs fields with the general tree-level 2HDM Lagrangian (2.7). Subsequently, the specific expressions for the couplings z_{12}, m_{12}^2, $\delta\lambda_{5,6,7}$, and $\delta\tilde{y}_{b,t,\tau}$ given in the previous section are inserted.

3.1 Vacuum expectation value v_d

We first derive a relation between the shift (counterterm) of the vev v_d and $\tan\beta$ and the MSSM parameter b, which feeds into all other Higgs couplings. The tadpole counterterm is determined by requiring that the one-loop H_d tadpole (1PI one-point function) vanishes:

$$\Gamma_{H_d} = t_d + \Delta t_d + [\Gamma_{H_d}]_{0,\text{heavy}} + [\Gamma_{H_d}]_{0,\text{light}} \overset{!}{=} 0. \tag{3.2}$$

The tadpole coupling t_d is given by (2.4) and set to zero by our renormalization convention. The tadpole counterterm in terms of the shifts of the vevs is also obtained from (2.4) and given by

$$\Delta t_d = \Delta \hat{m}_d^2 v_d + \hat{m}_d^2 \Delta v_d + \frac{1}{4}(\Delta g_1^2 + \Delta g_2^2)v_d(v_d^2 - v_u^2) - \Delta b v_u - b \Delta v_u$$
$$+ \frac{3}{4}(g_1^2 + g_2^2)v_d^2 \Delta v_d - \frac{1}{4}(g_1^2 + g_2^2)v_u^2 \Delta v_d - \frac{1}{2}(g_1^2 + g_2^2)v_d v_u \Delta v_u. \tag{3.3}$$

The (unrenormalized) SUSY loop contribution to Γ_{H_d} follows from the linear term in the 2HDM Lagrangian (2.7) after shifting the neutral Higgs fields by the vevs, which leads to

$$[\Gamma_{H_d}]_{0,\text{heavy}} = \delta m_{11}^2 v_d - \delta b v_u + \delta\lambda_1 v_d^3 - 3\delta\lambda_6 v_d^2 v_u + (\delta\lambda_3 + \delta\lambda_4 + \delta\lambda_5)v_d v_u^2 - \delta\lambda_7 v_u^3, \tag{3.4}$$

where δm_{11}^2, $\delta\lambda_{1-4}$ denote the SUSY loop contributions to the corresponding 2HDM parameters, which, however, will not be needed later on. The low-energy contributions can be neglected, as explained above, so the desired relation for Δv_d follows from $\Delta t_d + [\Gamma_{H_d}]_{0,\text{heavy}} \overset{!}{=} 0$.

This condition simplifies in the large-$\tan\beta$ limit. Applying $v_d \ll v_u$, $b \ll M_{\text{EW}}^2$, eq. (3.3) reads

$$\Delta t_d \approx \hat{m}_d^2 \Delta v_d - \Delta b v_u - \frac{1}{4}(g_1^2 + g_2^2)v_u^2 \Delta v_d \approx M_A^2 \Delta v_d - \Delta b \, v \tag{3.5}$$

The second approximation uses

$$g_1^2 = \frac{2(M_Z^2 - M_W^2)}{v^2}, \quad g_2^2 = \frac{2M_W^2}{v^2},$$
$$b = \frac{1}{2}M_A^2 \sin(2\beta),$$

$$\hat{m}_d^2 = \frac{1}{2}M_A^2 \sin(2\beta)\tan\beta - \frac{1}{2}M_Z^2 \cos(2\beta) \approx M_A^2 + \frac{M_Z^2}{2},$$
$$\hat{m}_u^2 = \frac{1}{2}M_A^2 \sin(2\beta)\cot\beta + \frac{1}{2}M_Z^2 \cos(2\beta) \approx -\frac{M_Z^2}{2},$$
$$v_d = v\cos\beta, \quad v_u = v\sin\beta \approx v, \tag{3.6}$$

which follow from inverting (2.4) for the renormalized parameters together with $t_u = t_d = 0$. Similarly,

$$[\Gamma_{H_d}]_{0,\text{heavy}} \approx -\delta b\, v - \delta\lambda_7 v^3. \tag{3.7}$$

Solving for Δv_d results in

$$\Delta v_d = \frac{(\delta b + \Delta b)v + \delta\lambda_7 v^3}{M_A^2}. \tag{3.8}$$

We see that unless the counterterm Δb is fine-tuned the shift of the vev is of order $M_{\text{SUSY}}^2/M_{\text{EW}}$, which is very large relative to its assumed value of order $M_{\text{EW}}/\tan\beta$. As discussed above, the shift of the vev is a finite quantity at leading order in large $\tan\beta$, since Δb cancels the divergences in δb. Similar statements now apply to the shift of $\tan\beta$. Inserting (3.8) into (2.6), we find

$$\Delta t_\beta \approx -\tan^2\beta\, \frac{\Delta v_d}{v} = -\tan^2\beta\, \frac{\delta b + \Delta b + \delta\lambda_7 v^2}{M_A^2} + O(\tan\beta). \tag{3.9}$$

The Higgs masses themselves receive no $\tan\beta$-enhanced one-loop corrections, so M_A corresponds to the physical mass of the CP-odd Higgs boson even at the one-loop level in our approximations.

3.2 Renormalization of $\tan\beta$

Contrary to the other parameters of the second set, there is no natural renormalization convention for the parameter $\tan\beta$. Gauge-independence and numerical stability of various schemes have been investigated in [5, 6]. Quite generally, we expect problems with stability of observables against adding perturbative corrections, if the finite part of a counterterm is large, since this leads to a large shift between the tree-level and one-loop corrected values of parameters and large counterterm contributions to observables.

From (3.9) we conclude that $\tan\beta$ receives large relative corrections, i.e. $\Delta t_\beta/\tan\beta \propto \tan\beta$, unless

$$\delta b + \Delta b + \delta\lambda_7 v^2 = O(1/\tan\beta). \tag{3.10}$$

Some of the stability features at large $\tan\beta$ discussed in [5, 6] can be understood on the basis of this equation. The $\overline{\text{DR}}$ scheme for $\tan\beta$ has $\Delta t_\beta = 0$ by definition and fulfills (3.10). So does the DCPR definition [11, 12], for which $\Delta t_\beta/\tan\beta \sim 1$, since it is given in terms of the difference of the diagonal field renormalization constants, which are not enhanced. (Alternatively, the shift of $\tan\beta$ is given in the DCPR scheme in terms of the unrenormalized $A^0 - Z^0$ mixing self-energy by the expression

$$\frac{\Delta t_\beta}{\tan\beta} = \frac{1}{M_Z \sin(2\beta)}\,\text{Re}\,\Sigma_{A^0 Z^0}(M_A^2). \tag{3.11}$$
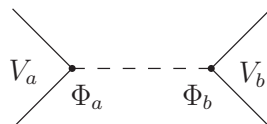
Figure 4: Virtual Higgs boson exchange.

The explicit result given in the appendix of [11] shows that $\Sigma_{A^0 Z^0}(M_A^2) = O(1/\tan\beta)$ for $M_{\text{SUSY}} \gg M_{\text{EW}}$ and counting $m_{b,\tau} \sim M_{\text{EW}}/\tan\beta$. It follows that (3.11) is consistent with $\Delta t_\beta/\tan\beta \propto 1$.) Indeed, these two schemes were found to exhibit good stability. On the other hand, the m_3-scheme ($\overline{\text{DR}}$ for the b parameter), implies

$$\frac{\Delta t_\beta}{\tan\beta} \approx -\tan\beta \frac{[\delta b]_{\text{fin}} + \delta\lambda_7 v^2}{M_A^2}, \tag{3.12}$$

which is very large, of order $M_{\text{SUSY}}^2/M_{\text{EW}}^2 \times \tan\beta$. The Higgs mass and tadpole scheme [5] also violate (3.10), and are unstable.

Good perturbative renormalization schemes should thus have $\delta b + \Delta b + \delta\lambda_7 v^2 = 0$ in the large-$\tan\beta$ limit. Since $b \sim M_{\text{EW}}^2/\tan\beta$ but $\delta b \sim M_{\text{SUSY}}^2$, this implies fine-tuning of the counterterm for the b parameter as expected.

3.3 Physical Higgs fields

To determine the physical Higgs fields, we first bring the kinetic terms of the 2HDM (2.7) into canonical form by a field redefinition. This is not necessary to determine the Higgs masses, which receive no $\tan\beta$-enhanced corrections, since there are no tree-level suppressions, or observables that involve only highly virtual internal Higgs lines, as for instance in low-energy flavour physics. The latter statement can be illustrated with the help of figure 4, representing part of a diagram with an internal Higgs propagator. The expression for this diagram using the Lagrangian (2.7) is $V_b^\dagger i [(Z q^2 - m^2)^{-1}]_{ba} V_a$, where the propagator is a 2×2 matrix in Higgs doublet space. In low-energy scattering the momentum transfer satisfies $q^2 \ll m^2$, so the term involving the kinetic-mixing matrix Z_{ab} in the 2HDM Lagrangian drops out at leading order in the low-energy expansion, and cannot affect physical observables. If one performs a field rotation to diagonalize Z_{ab}, the rotation affects the vertices V_a and mass matrix m_{ab}^2 in such a way, that once again the effect disappears at leading order in q^2/m^2. However, in this paper our aim is to calculate the couplings of on-shell Higgs bosons ($q^2 \sim m^2$) to quarks and the Higgs fields themselves, relevant to Higgs production and decay, so we need the fields corresponding to the physical states.

The field redefinition that diagonalizes the kinetic terms (and includes the shift of the fields by the vevs (2.2)) is

$$\Phi_1^1 = H_d^1 \to [v_d]_0 + \left(1 - \frac{z_{11}}{2}\right) H_d^0 + \frac{1+a}{2} z_{12} H_u^{0*},$$
$$\Phi_1^2 = H_d^2 \to \left(1 - \frac{z_{11}}{2}\right) H_d^- - \frac{1+a}{2} z_{12} H_u^{+*},$$

$$-\Phi_2^{1*} = H_u^2 \rightarrow [v_u]_0 + \left(1 - \frac{z_{22}}{2}\right) H_u^0 + \frac{1-a}{2} z_{12} H_d^{0*},$$

$$\Phi_2^{2*} = H_u^1 \rightarrow \left(1 - \frac{z_{22}}{2}\right) H_u^+ - \frac{1-a}{2} z_{12} H_d^{-*}, \tag{3.13}$$

where z_{ab} is the one-loop correction to Z_{ab}, and we used that the hermitian matrix Z_{ab} is also symmetric due to CP-conservation. The diagonal coefficients z_{11}, z_{22} can be set to zero, since they are ordinary one-loop corrections to a non-vanishing tree term. The interesting terms are those that mix H_d with the complex conjugate of H_u. The arbitrary quantity a parameterizes a real field rotation in (Φ_1, Φ_2) space, which preserves the diagonal form of the kinetic term. We could set $a = 0$, but prefer to keep it to demonstrate explicitly the independence of physical quantities on a below. Note that we do not rotate the fields and then shift them by the vevs, since the vevs (and $\tan\beta$) have been defined as parameters of the MSSM Lagrangian before matching to the 2HDM.

After substituting (3.13) into (2.7), we perform a unitary (in fact orthogonal, on account of CP-conservation) field rotation to diagonalize the Higgs mass matrix. The transformation to the physical Higgs fields $h^0, H^0, A^0, H^\pm$, including the pseudo-Goldstone fields $G^0, G^\pm$, is

$$\begin{pmatrix} \mathrm{Im}\, H_u^0 \\ \mathrm{Im}\, H_d^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} s_\beta + \delta s_\beta & c_\beta + \delta c_\beta \\ -[c_\beta + \delta c_\beta] & s_\beta + \delta s_\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix},$$

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = \begin{pmatrix} s_\beta + \delta s_\beta & c_\beta + \delta c_\beta \\ -[c_\beta + \delta c_\beta] & s_\beta + \delta s_\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix},$$

$$\begin{pmatrix} \mathrm{Re}\, H_u^0 \\ \mathrm{Re}\, H_d^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_\alpha + \delta c_\alpha & s_\alpha + \delta s_\alpha \\ -[s_\alpha + \delta s_\alpha] & c_\alpha + \delta c_\alpha \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix}, \tag{3.14}$$

where $\delta s_\beta, \delta c_\beta, \delta s_\alpha, \delta c_\alpha$ parameterize the correction to the corresponding MSSM tree-level rotation, and we use the conventional notation $s_\phi \equiv \sin\phi$, $c_\phi \equiv \cos\phi$. We already incorporated here that the correction δc_β to the tree-level mixing matrix turns out to be the same for the CP-odd and the charged Higgs fields. The mixing angle α is given by

$$\tan 2\alpha = \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} \tan 2\beta. \tag{3.15}$$

The correction terms δs_β, δc_β are of the size of an ordinary loop correction, and hence relevant only if the corresponding tree contribution is suppressed. This is the case for the off-diagonal elements, since $c_\beta \propto 1/\tan\beta$. We therefore neglect the δs_β terms relative to $s_\beta \approx 1$. For the off-diagonal correction we obtain

$$\delta c_\beta = -\frac{1+a}{2} z_{12} + \frac{\delta b + \Delta b + \delta\lambda_7 v^2}{M_A^2}. \tag{3.16}$$

The second term vanishes in "good" renormalization schemes.

In determining the correction to α, the cases $M_A > M_Z$ and $M_Z > M_A$ should be distinguished. In the following we discuss explicitly only the case $M_A > M_Z$. The other case follows roughly (that is, up to some signs) from interchanging h^0 and H^0. For large

$\tan \beta$ we have $\tan 2\beta \approx -2/\tan \beta$, so (3.15) implies that either c_α or s_α is small, unless M_A is very close to M_Z to compensate the $\tan \beta$ suppression. For $M_A > M_Z$ the relevant case is s_α small and negative and c_α near 1. The correction to $\tan 2\alpha$ is (valid for both cases $M_A > M_Z$, and $M_A < M_Z$)

$$
\begin{aligned}
\delta t_{2\alpha} = a z_{12} + \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} z_{12} \\
- \frac{2}{M_A^2} \left(\frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} \left[\delta b + \Delta b + \delta \tilde{\lambda}_7 v^2 \right] + \frac{2 M_A^2}{M_A^2 - M_Z^2} \delta \tilde{\lambda}_7 v^2 \right).
\end{aligned} \tag{3.17}
$$

For $M_A > M_Z$ we have $\delta s_\alpha = \frac{1}{2}\delta t_{2\alpha}$, $\delta c_\alpha \approx 0$ (relative to $c_\alpha \approx 1$). The field redefinitions that lead to the mass eigenstates therefore read

$$
\begin{aligned}
H_u^0 &= \frac{1}{\sqrt{2}} \left(c_\alpha h^0 + [s_\alpha + \delta s_\alpha] H^0 + i[c_\beta + \delta c_\beta] A^0 + i s_\beta G^0 \right), \\
H_d^0 &= \frac{1}{\sqrt{2}} \left(-[s_\alpha + \delta s_\alpha] h^0 + c_\alpha H^0 + i s_\beta A^0 - i[c_\beta + \delta c_\beta] G^0 \right), \\
H_d^- &= s_\beta H^- - [c_\beta + \delta c_\beta] G^-, \\
H_u^+ &= [c_\beta + \delta c_\beta] H^+ + s_\beta G^+.
\end{aligned} \tag{3.18}
$$

We briefly comment on the case $M_A = M_Z$, since (3.17) appears to be singular for this parameter input. The masses used here are tree-level masses, since we keep loop corrections only when they are enhanced by $\tan \beta$ relative to the tree term. When $M_A = M_Z$ the CP-even Higgs bosons h^0, H^0 are mass-degenerate and in fact physically indistinguishable. Eq. (3.17) is derived under the assumption that the difference between the diagonal elements of the neutral Higgs boson mass matrix is not small. Otherwise one must keep the one-loop correction to the diagonal elements in the calculation of $\delta t_{2\alpha}$, and the approximation discussed here cannot be used. Thus the resonant enhancement in (3.17) is a real effect as long as $M_A^2 - M_Z^2$ is large compared to $M_{\rm EW}^2 \times$ loop factor. However, if the widths of the Higgs bosons are so large that the h^0 and H^0 intermediate states are not separated by some measurement, nothing distinguishes the large tree-unsuppressed $c_\alpha H^0$ component from the smaller $[s_\alpha + \delta s_\alpha] h^0$ component in (3.18) and there is no observable $\tan \beta$-enhancement in this measurement. In the following, we do not consider this degenerate case further.

3.4 Yukawa couplings

The Yukawa couplings and quark mass terms follow from inserting (3.18) into (2.7). The bottom mass term is of special interest. The relevant bilinear terms are

$$
\mathcal{L}_{b,\text{mass}} = - \left[(y_b + \delta y_b + \Delta y_b)(v_d + \Delta v_d) + \delta \tilde{y}_b (v_u + \Delta v_u) \right] \bar{b}_R b_L + \text{h.c.} \tag{3.19}
$$

The physical (pole) quark mass m_b is obtained from this mass term plus low-energy self-energy contributions calculated in the 2HDM. As before we keep one-loop corrections only when they are enhanced relative to the tree expressions. This allows us to neglect the low-energy contributions (see [13, 14] for the standard model electroweak loops) as well as Δv_u, and δy_b in (3.19). Δy_b is the counterterm that cancels the divergences from superpartner

loops in δy_b. The finite part of Δy_b defines the relation between the physical bottom quark mass and the renormalized Yukawa coupling y_b. There is no reason to artificially define this finite part to be parametrically larger than the one of δy_b, so we may drop Δy_b as well. Introducing the definitions

$$\epsilon_t = \frac{\delta\tilde{y}_t}{y_t}, \quad \epsilon_b = \frac{\delta\tilde{y}_b}{y_b}, \quad \epsilon_\tau = \frac{\delta\tilde{y}_\tau}{y_\tau} \tag{3.20}$$

we obtain the mass terms

$$\mathcal{L}_{\text{mass}} = -m_t \bar{t}_R t_L - m_b \bar{b}_R b_L - m_\tau \bar{\tau}_R \tau_L + \text{h.c.} \tag{3.21}$$

with

$$\begin{aligned} m_t &= y_t v_u = y_t v s_\beta, \\ m_b &= y_b(v_d + \Delta v_d) + \delta\tilde{y}_b v_u = y_b v s_\beta \left(\frac{1}{\tan\beta + \Delta t_\beta} + \epsilon_b \right). \end{aligned} \tag{3.22}$$

Note that we do not expand in Δt_β, since according to (3.9) it counts as a $\tan\beta$-enhanced loop correction relative to $\tan\beta$, just as $\epsilon_b \tan\beta$ does relative to 1. No such enhancements are present for the top quark mass, which retains its tree expression. The expression for m_τ is obtained from the one for m_b by replacing $(y_b, \epsilon_b) \to (y_\tau, \epsilon_\tau)$.

Solving (3.22) for y_b and eliminating $y_{b,t}$ by $m_{b,t}$, as it is conventionally done, we obtain the Higgs-Yukawa interactions

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} = -\frac{m_t}{\sqrt{2}v} &\Bigg\{ \left(\frac{1}{\tan\beta} + \epsilon_t + \delta c_\beta - \frac{1-a}{2}z_{12} \right) \left(i\bar{t}_R t_L A^0 - \sqrt{2}\bar{t}_R b_L H^+ \right) \\ &+ \frac{c_\alpha}{s_\beta}\bar{t}_R t_L h^0 + \frac{s_\alpha + \delta s_\alpha - \epsilon_t + \frac{1-a}{2}z_{12}}{s_\beta}\bar{t}_R t_L H^0 \\ &+ i\bar{t}_R t_L G^0 - \sqrt{2}\bar{t}_R b_L G^+ \Bigg\} \\ -\frac{m_b\left[\tan\beta + \Delta t_\beta\right]}{\sqrt{2}v\left(1 + \epsilon_b\left[\tan\beta + \Delta t_\beta\right]\right)} &\Bigg\{ i\bar{b}_R b_L A^0 - \sqrt{2}\bar{b}_R t_L H^- \\ &- \frac{s_\alpha + \delta s_\alpha - \epsilon_b - \frac{1+a}{2}z_{12}}{s_\beta}\bar{b}_R b_L h^0 + \frac{c_\alpha}{s_\beta}\bar{b}_R b_L H^0 \\ &- \left(\frac{1}{\tan\beta} + \epsilon_b + \delta c_\beta + \frac{1+a}{2}z_{12} \right) \left(i\bar{b}_R b_L G^0 - \sqrt{2}\bar{b}_R t_L G^- \right) \Bigg\} \\ +\text{h.c.}, & \end{aligned} \tag{3.23}$$

where as before we neglect ordinary one-loop corrections relative to unsuppressed tree terms, and the τ Yukawa couplings are given by the bottom ones after replacing all bottom parameters and fields by the corresponding expressions for tau. Several comments can be made. First, the explicit dependence on the arbitrary parameter a cancels with the implicit dependence in (3.16), (3.17). Second, our result is independent on the renormalization

convention for $\tan\beta$. A change of renormalization prescription is effected by a change in the finite part of Δt_β, or (3.9) Δv_d, or (3.8) Δb. From (3.16), (3.17), we therefore find that

$$[\tan\beta + \Delta t_\beta], \left[\frac{1}{\tan\beta} + \delta c_\beta\right], [s_\alpha + \delta s_\alpha], s_\beta, c_\alpha \tag{3.24}$$

are invariant under a change of renormalization prescription that contains only $\tan\beta$-enhanced one-loop terms. (For $[\tan\beta + \Delta t_\beta]$ this can be seen directly from (2.5), since it equals the scheme-independent bare quantity $[\tan\beta]_0$. Of course the bare quantity $[\tan\beta]_0$ is finite only at the level of the $\tan\beta$-enhanced terms.).

Eq. (3.23) shows that a resummation of $\tan\beta$-enhanced terms is necessary only in the overall factor multiplying the bottom (and tau) Yukawa couplings [3], where the need for resummation arises from using the quark (lepton) mass as a parameter rather than the MSSM Yukawa coupling. All of the Yukawa couplings which are $\tan\beta$-suppressed at tree-level receive unsuppressed loop corrections, which are incorporated in (3.23) through the "wrong-Higgs" Yukawa couplings $\epsilon_{t,b,\tau}$, and the quantities δc_β, δs_α, z_{12} from the one-loop effects in the Higgs sector. The loop-induced couplings may well exceed the tree terms, if $\tan\beta$ is large.

To compare our result with previous results in the literature, see eq. (21) in [3], we consider the bottom quark coupling to the lightest Higgs boson h^0, which is the product of two factors

$$\frac{m_b\,[\tan\beta + \Delta t_\beta]}{\sqrt{2}v\,(1 + \epsilon_b\,[\tan\beta + \Delta t_\beta])} \times \frac{s_\alpha + \delta s_\alpha - \epsilon_b - \frac{1+a}{2}z_{12}}{s_\beta}. \tag{3.25}$$

With respect to the first factor, note that we do not discuss logarithmic one-loop effects, and hence do not distinguish the physical quark mass m_b from the running $\overline{\text{MS}}$ quark mass $\overline{m}_b(Q)$ used in [3]. In reality, renormalization group evolution from m_b to the electroweak scale is an important effect that should be taken into account as done in [3]. The main difference in the first factor constitutes the presence of the (finite) counterterm Δt_β, which must not be neglected in general, and which renders the resummed result manifestly scheme-independent. This clarifies that the previously known result where $\Delta t_\beta = 0$, is only valid in "good" renormalization schemes (3.10) where the one-loop shift of $\tan\beta$ is not itself $\tan\beta$-enhanced, as also observed recently in the related context of the muon Yukawa coupling [15]. The resummation formula from [3] without the counterterm contribution should not be used in other schemes, for which Δt_β is an important correction in the first factor.

Using $s_\beta \approx 1$ in the one-loop corrections, and (3.17), the second factor can be rewritten as

$$\frac{s_\alpha}{s_\beta} - \epsilon_b + \frac{M_Z^2}{M_A^2 - M_Z^2}\,z_{12} - \frac{2\delta\lambda_7 v^2}{M_A^2 - M_Z^2} - \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}\,\frac{\delta b + \Delta b + \delta\lambda_7 v^2}{M_A^2}. \tag{3.26}$$

The first two terms reproduce the corresponding result in [3], but the Higgs-sector effects embodied in the last three terms have not been included there. Higgs-sector effects related to kinetic mixing have recently been computed in [7]. The $\overline{\text{DR}}$ scheme for $\tan\beta$ is assumed there, hence the last term involving $\delta b + \Delta b + \delta\lambda_7 v^2 \propto \Delta t_\beta = 0$ vanishes. We find several discrepancies in the comparison with [7]. The third term proportional to z_{12} would be consistent with the result of [7] if $z_{12} = \epsilon_{\text{GP}}$, but our result for z_{12} in (2.9) gives $z_{12} = -\epsilon_{\text{GP}}$.

We also find that the corrections to the $\bar{t}_L t_R H^0$ coupling are not included in [7]. The most important difference is that the fourth term involving $\delta\lambda_7$ is missed, since it is assumed that Higgs-sector effects arise only from kinetic mixing, and not (also) from the Higgs self-couplings. The large quantum correction to the tree-level suppressed charged Higgs $\bar{t}_R b_L H^+$ coupling is present in the discussion of the $b \to s\gamma$ transition in [16, 17]. These papers account for the gluino contribution to ϵ_t. The Higgs-sector terms involving δc_β and z_{12} are again not considered.

More precisely, in the expression for the third term proportional to z_{12} ref. [7] has $M_{h^0}^2/(M_{H^0}^2 - M_{h^0}^2)$, where we have $M_Z^2/(M_A^2 - M_Z^2)$. The two expressions are consistent with each other, since in the one-loop correction we may use the large-$\tan\beta$ limits of the tree-level mass relations, which give $M_{h^0} = M_Z$, and $M_{H^0} = M_A$. In reality, the Higgs masses must receive large quantum corrections to satisfy experimental limits, which are not included in our equations, since they are not $\tan\beta$-enhanced. Since the apparent singularity for $M_A = M_Z$ arises from the degeneracy of the two CP-even Higgs bosons, as discussed above, it is plausible that $M_{h^0}^2/(M_{H^0}^2 - M_{h^0}^2)$ provides a better approximation, even though formally the difference to $M_Z^2/(M_A^2 - M_Z^2)$ is a higher-order effect.

In section 4 we perform some representative numerical estimates of the new Higgs-sector corrections to the Yukawa couplings and compare them to the previously known term ϵ_b.

3.5 Higgs self-couplings

The Higgs self-couplings are obtained in the same straightforward fashion. Here we focus on the trilinear couplings, since the quartic self-interactions with coefficients induced by quantum effects will be too small to be measured in any foreseeable experiment. The detection of $\tan\beta$- or loop-suppressed trilinear couplings provides a challenge even to the ILC (see, e.g., [18]).

The Lagrangian for the trilinear couplings

$$\mathcal{L}_{HHH} = \mathcal{L}_{HHH,\,\text{large}} + \mathcal{L}_{HHH,\,\text{small}} \tag{3.27}$$

consists of a piece $\mathcal{L}_{HHH,\,\text{large}}$ involving interactions that do not vanish at tree level when $\tan\beta \to \infty$, and another piece $\mathcal{L}_{HHH,\,\text{small}}$ with interactions that do. The tree-level couplings in the MSSM are standard and will not be repeated here. We find a compact expression for the $\tan\beta$-enhanced one-loop corrections relative to the small tree-level interactions in $\mathcal{L}_{HHH,\,\text{small}}$, applicable to the case $M_A > M_Z$:

$$
\begin{aligned}
\mathcal{L}_{HHH,\,\text{small}}^{\text{one-loop}} = -\frac{v}{\sqrt{2}} \bigg\{ & (-1)(3M_A^2 + 2M_Z^2)\, C_H\, h^{0^2} H^0 \\
& + (M_A^2\, C_H - \delta\lambda_6 - \delta\lambda_7)\,(H^{0^2} + A^{0^2})H^0 \\
& + 2\left[(M_A^2 - 2M_W^2)\, C_H - (\delta\lambda_6 + \delta\lambda_7)\right] H^0 H^+ H^- \\
& + 2\,(M_A^2 - M_Z^2)\, C_H\, h^0 A^0 G^0 - 2\delta\lambda_5\, H^0 A^0 G^0 - M_A^2\, C_H\, H^0(G^{0^2} + 2\,G^+ G^-) \\
& + 2\,(M_A^2 - M_Z^2 + M_W^2)\, C_H\, h^0(H^- G^+ + H^+ G^-) \bigg\},
\end{aligned}
\tag{3.28}
$$

where

$$C_H = \frac{1}{M_A^2 - M_Z^2} \left(\frac{M_Z^2}{v^2} \left[\frac{\delta b + \Delta b + \delta \lambda_7 v^2}{M_A^2} - \frac{z_{12}}{2} \right] + \delta \lambda_7 \right). \qquad (3.29)$$

4. Numerical estimates

In this section we present numerical estimates of the Higgs-sector corrections by evaluating them for degenerate SUSY masses, and by performing a random parameter space scan. We are especially interested in comparing the new Higgs-sector effects to the known one-loop induced "wrong-Higgs" Yukawa couplings.

4.1 Degenerate SUSY masses

We begin the analysis of the size of the new terms in the Yukawa couplings relative to $\epsilon_{b,t}$ by assuming that all SUSY parameters are equal to a common mass scale M_{SUSY} and by allowing for arbitrary signs in the μ and A parameters, but not the gaugino masses. In the following we give analytic results for $\epsilon_{b,t,\tau}$ and the quantities z_{12} and $\delta \lambda_{5-7}$, which determine the Higgs-sector corrections to the Yukawa couplings and the Higgs self-couplings, and numerical results to judge the importance of individual terms. To make contact with the literature, we introduce the rescaled A parameters $\widetilde{A}$ by $A_i = y_i \widetilde{A}_i$. In units of $\mathrm{sgn}(\mu)/(96\pi^2)$, we find

$$\epsilon_b = 3y_t^2 \, \mathrm{sgn}(\widetilde{A}_t) + 32\pi\alpha_s - 18\pi\alpha_{\mathrm{em}} \left(\frac{1}{s_w^2} + \frac{11}{27 c_w^2} \right)$$

$$= 9.65 \, + \, \mathrm{sgn}(\widetilde{A}_t) \frac{3.02}{s_\beta^2},$$

$$\epsilon_t = -3y_b^2 \, \mathrm{sgn}(\widetilde{A}_b) - 32\pi\alpha_s + 18\pi\alpha_{\mathrm{em}} \left(\frac{1}{s_w^2} + \frac{5}{27 c_w^2} \right)$$

$$= -9.77 \, - \, \mathrm{sgn}(\widetilde{A}_b) \frac{2.14}{[50 \, c_\beta]^2},$$

$$\epsilon_\tau = -18\pi\alpha_{\mathrm{em}} \left(\frac{1}{s_w^2} - \frac{1}{3 c_w^2} \right) = -1.80 \qquad (4.1)$$

for the "wrong-Higgs" Yukawa couplings, and

$$z_{12} = 3y_t^2 \, \mathrm{sgn}(\widetilde{A}_t) + 3y_b^2 \, \mathrm{sgn}(\widetilde{A}_b) + y_\tau^2 \, \mathrm{sgn}(\widetilde{A}_\tau) + 4\pi\alpha_{\mathrm{em}} \left(\frac{3}{s_w^2} + \frac{1}{c_w^2} \right)$$

$$= 1.45 \, + \, \mathrm{sgn}(\widetilde{A}_t) \frac{3.02}{s_\beta^2} \, + \, \mathrm{sgn}(\widetilde{A}_b) \frac{2.14}{[50 \, c_\beta]^2} \, + \, \mathrm{sgn}(\widetilde{A}_\tau) \frac{0.26}{[50 \, c_\beta]^2},$$

$$\delta \lambda_5 = \mathrm{sgn}(\mu) \left\{ -3y_t^4 - 3y_b^4 - y_\tau^4 + 16\pi^2\alpha_{\mathrm{em}}^2 \left(\frac{3}{s_w^4} + \frac{1}{c_w^4} + \frac{2}{c_w^2 s_w^2} \right) \right\}$$

$$= \mathrm{sgn}(\mu) \left\{ 0.712 - \frac{3.04}{s_\beta^4} - \frac{1.59}{[50 \, c_\beta]^4} \right\},$$

$$\delta \lambda_6 = 15 \, y_b^4 \, \mathrm{sgn}(\widetilde{A}_b) - 3y_t^4 \, \mathrm{sgn}(\widetilde{A}_t) + 5y_\tau^4 \, \mathrm{sgn}(\widetilde{A}_\tau)$$

$$- \frac{3\pi\alpha_{\mathrm{em}}}{c_w^2 \, s_w^2} \left(-3y_t^2 \, \mathrm{sgn}(\widetilde{A}_t) + 3y_b^2 \, \mathrm{sgn}(\widetilde{A}_b) + y_\tau^2 \, \mathrm{sgn}(\widetilde{A}_\tau) \right)$$

$$+ 32\pi^2 \alpha_{\text{em}}^2 \left(\frac{3}{s_w^4} + \frac{1}{c_w^4} + \frac{2}{s_w^2 c_w^2} \right)$$

$$= 1.42 - \text{sgn}(\widetilde{A}_t) \left(\frac{3.04}{s_\beta^4} - \frac{1.29}{s_\beta^2} \right) - \text{sgn}(\widetilde{A}_b) \left(\frac{0.91}{[50\, c_\beta]^2} - \frac{7.62}{[50\, c_\beta]^4} \right)$$

$$- \text{sgn}(\widetilde{A}_\tau) \left(\frac{0.11}{[50\, c_\beta]^2} - \frac{0.34}{[50\, c_\beta]^4} \right),$$

$$\delta\lambda_7 = 15\, y_t^4\, \text{sgn}(\widetilde{A}_t) - 3y_b^4\, \text{sgn}(\widetilde{A}_b) - y_\tau^4\, \text{sgn}(\widetilde{A}_\tau)$$

$$+ \frac{3\pi\alpha_{\text{em}}}{c_w^2\, s_w^2} \left(-3y_t^2\, \text{sgn}(\widetilde{A}_t) + 3y_b^2\, \text{sgn}(\widetilde{A}_b) + y_\tau^2\, \text{sgn}(\widetilde{A}_\tau) \right)$$

$$+ 32\pi^2 \alpha_{\text{em}}^2 \left(\frac{3}{s_w^4} + \frac{1}{c_w^4} + \frac{2}{s_w^2 c_w^2} \right)$$

$$= 1.42 + \text{sgn}(\widetilde{A}_t) \left(\frac{15.2}{s_\beta^4} - \frac{1.29}{s_\beta^2} \right) + \text{sgn}(\widetilde{A}_b) \left(\frac{0.91}{[50\, c_\beta]^2} - \frac{1.52}{[50\, c_\beta]^4} \right)$$

$$+ \text{sgn}(\widetilde{A}_\tau) \left(\frac{0.11}{[50\, c_\beta]^2} - \frac{0.07}{[50\, c_\beta]^4} \right) \tag{4.2}$$

for the Higgs-sector couplings. The numerical values in (4.2) have been evaluated with SM parameter input as follows: for the gauge boson masses we choose $M_W = 80.398\,\text{GeV}$ and $M_Z = 91.188\,\text{GeV}$, the gauge couplings are fixed to $\alpha_{\text{em}} = \alpha_{\text{em}}^{\overline{\text{MS}}}(M_Z) = 1/127.9$ and $\alpha_s = \alpha_s^{\overline{\text{MS}}}(M_Z) = g_3^2/(4\pi) = 0.118$. The cosine of the weak mixing angle is given by $c_w = M_W/M_Z$. For the top Yukawa coupling we use $y_t = \overline{m}_t/(vs_\beta) \simeq 1.003/s_\beta$, where $\overline{m}_t = 171.7\,\text{GeV}$ and $v = s_w M_W/\sqrt{2\pi\alpha_{\text{em}}} \simeq 171.2\,\text{GeV}$. For the down-type and lepton Yukawa couplings we here use the tree-level relations $y_b = \overline{m}_b/(vc_\beta) \simeq 0.0169/c_\beta$ and $y_\tau = \overline{m}_\tau/(vc_\beta) \simeq 0.0102/c_\beta$, where $\overline{m}_b = 2.89\,\text{GeV}$ and $\overline{m}_\tau = 1746.24\,\text{MeV}$. All fermion masses correspond to the renormalized masses in the $\overline{\text{MS}}$ scheme at $\mu = M_Z$ [19] rather than the pole masses.

Since ϵ_b is nearly independent of $\tan\beta$ for $\tan\beta \gg 1$, we show the $\tan\beta$ dependence of the ratios z_{12}/ϵ_b and $\delta\lambda_{5-7}/\epsilon_b$ from (4.1), (4.2) in figure 5 for different choices of the signs of the $\widetilde{A}$-parameters. The slepton terms give a negligible contribution except for z_{12} at very large values of $\tan\beta$, so we always set the sign of $\widetilde{A}_\tau$ to +1. For $\tan\beta$ smaller than 20–30 the value of z_{12}, $\delta\lambda_{5-7}$ is determined by the $\widetilde{A}_t$ terms and the gaugino contributions. For $\delta\lambda_7$, in particular, which feeds into the Yukawa couplings, the terms proportional to $\widetilde{A}_t$ give by far the largest contributions, while the effect of $\widetilde{A}_b$ becomes relevant for very large $\tan\beta$ values, reaching up to a 20% contribution. On the other hand we observe from the plots that the $\widetilde{A}_b$ terms and a change in the sign of $\widetilde{A}_b$ has a large effect on $\delta\lambda_6$, which is driven by the $\text{sgn}(\widetilde{A}_b)/c_\beta^4$ term in (4.2) with its sizeable coefficient. In general, under the assumption of a common mass scale for all SUSY parameters, we thus conclude that the quantities z_{12}, $\delta\lambda_{5,6}$ and particularly $\delta\lambda_7$ are comparable in size with the "wrong-Higgs" couplings $\epsilon_{b,t}$ even though they do not receive gluino contributions. Note that $|\delta\lambda_7|$ is larger than $|\epsilon_{b,t}|$ for all choices of signs of the $\widetilde{A}_{t,b}$.

4.2 Estimates using random parameter-space sampling

We now investigate in more detail the size of the Higgs one-loop corrections to the Yukawa
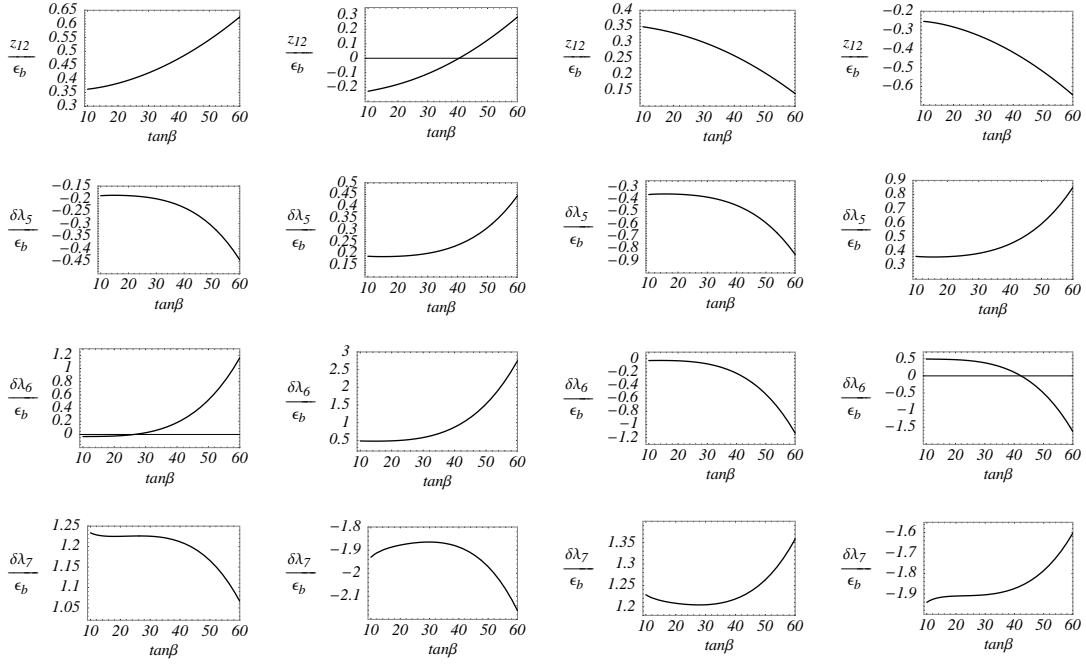
Figure 5: Dependence of Higgs-sector couplings, normalized to ϵ_b, on $\tan\beta$. For the ratios z_{12}/ϵ_b, $\delta\lambda_{6,7}/\epsilon_b$ the plots correspond to the sign choices $\{\mathrm{sgn}(\widetilde{A}_t), \mathrm{sgn}(\widetilde{A}_b)\}$ = $\{+,+\}, \{-,+\}, \{+,-\}, \{-,-\}$. In the plots for $\delta\lambda_5/\epsilon_b$ we adopt $\{\mathrm{sgn}(\mu), \mathrm{sgn}(\widetilde{A}_t)\}$ = $\{+,+\}, \{-,+\}, \{+,-\}, \{-,-\}$.

interactions relative to $\epsilon_{b,t}$ for different SUSY parameter input. We consider good renormalization schemes for $\tan\beta$, i.e. schemes satisfying $\delta b + \Delta b + \delta\lambda_7 v^2 = 0$ in the large-$\tan\beta$ limit, in which case the two relevant corrections from (3.23) are

$$ h_1 \equiv z_{12}, \qquad h_2 \equiv \frac{M_Z^2 z_{12} - 2v^2 \delta\lambda_7}{M_A^2 - M_Z^2} \,. \tag{4.3} $$

In terms of the quantities $h_{1,2}$, the one-loop improved couplings of the Higgs fields to the bottom and top quark are rewritten as, omitting the global factors outside the curly brackets in (3.23),

$$
\begin{aligned}
\bar{b}_R b_L h^0 : && \frac{s_\alpha}{s_\beta} + \delta s_\alpha - \epsilon_b - \frac{1+a}{2} z_{12} &= \frac{s_\alpha}{s_\beta} - \epsilon_b + h_2 \,, \\
i\bar{b}_R b_L G^0 : && \frac{1}{\tan\beta} + \epsilon_b + \delta c_\beta + \frac{1+a}{2} z_{12} &= \frac{1}{\tan\beta} + \epsilon_b \,, \\
i\bar{t}_R t_L A^0 : && \frac{1}{\tan\beta} + \epsilon_t + \delta c_\beta - \frac{1-a}{2} z_{12} &= \frac{1}{\tan\beta} + \epsilon_t - h_1 \,, \\
\bar{t}_R t_L H^0 : && \frac{s_\alpha}{s_\beta} + \delta s_\alpha - \epsilon_t + \frac{1-a}{2} z_{12} &= \frac{s_\alpha}{s_\beta} - \epsilon_t + h_1 + h_2 \,.
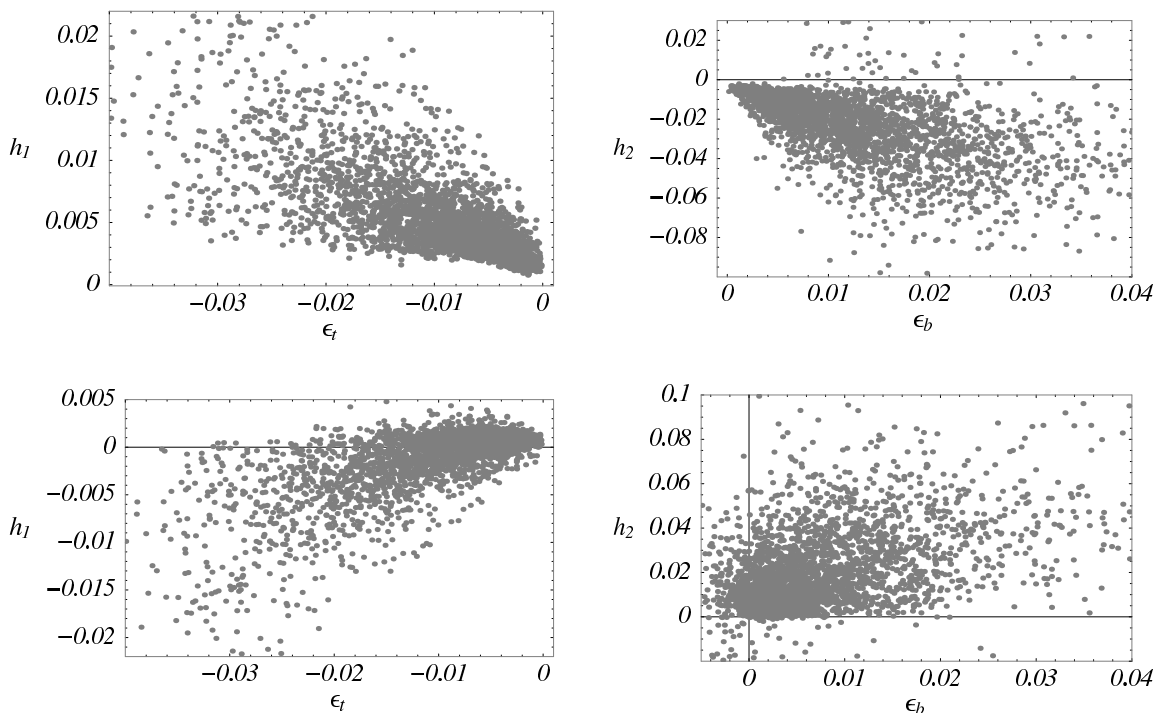\end{aligned}
\tag{4.4}
$$

Figure 6: The corrections h_1 and h_2 versus ϵ_t and ϵ_b respectively, for $M_A = 200\,\text{GeV}$ and positive μ. The plots in the first row have $\text{sgn}(\widetilde{A}_t) = +1$ and those in the second row have $\text{sgn}(\widetilde{A}_t) = -1$ ($\text{sgn}(\widetilde{A}_b) = +1$ in all plots).

The function C_H, which appears in the one-loop corrections to the small tree-level trilinear Higgs couplings (3.28), also has a simple expression in terms of h_2, which is given by

$$C_H = -\frac{h_2}{2v^2}\,. \tag{4.5}$$

We perform a random parameter sampling for the SUSY parameters in the following ranges: $500\,\text{GeV} \le \tilde{m}_i \le 5\,\text{TeV}$, with $\tilde{m}_i$ being the squark, slepton or gaugino masses, and $500\,\text{GeV} \le |\mu|, |\widetilde{A}_t|, |\widetilde{A}_b|, |\widetilde{A}_\tau| \le 5\,\text{TeV}$. The different sign assignments for the parameters μ, $\widetilde{A}_t$ and $\widetilde{A}_b$ are explored separately, but we fix $\text{sgn}(\widetilde{A}_\tau) = +1$. Since the M_A dependence of h_2 comes in only through the prefactor $1/(M_A^2 - M_Z^2)$, we shall fix it to the reference value $M_A = 200\,\text{GeV}$. Here contrary to the analytic expressions in the previous subsection, we correctly include the $\tan\beta$-enhanced loop correction to the down-type quark and lepton Yukawa couplings, i.e. $y_{b,\tau} = \overline{m}_{b,\tau}/(vc_\beta) \times 1/(1 + \epsilon_{b,\tau}\tan\beta)$ is used. We remark that not all points allowed in the parameter scan are physical, since vacuum stability requirements do not allow large A terms relative to the scale of the SUSY particle masses.

Let us first study the case of positive μ, where ϵ_b is preferentially positive, so that y_b is reduced by the one-loop correction. In figure 6 we show h_1 (left) and h_2 (right) versus ϵ_t and ϵ_b, respectively, for both signs of $\tilde{A}_t$ (positive in the upper plots, negative in the lower ones). For positive μ, the value of $\tan\beta$ does not significantly change the shape of the scatter plot, and hence has been fixed to $\tan\beta = 35$. The typical size of the effective

couplings is a few percent. Values of $|h_2|$ range up to 0.1, which implies that the one-loop induced coupling may exceed the $\tan\beta$-suppressed tree coupling for values of $\tan\beta$ as small as 10. In comparison, h_1, which originates only from kinetic mixing, tends to be significantly smaller. Thus, the Higgs sector correction h_2 competes in size with the "wrong-Higgs" Yukawa coupling ϵ_b even for large values of M_A, and is in fact the dominant one-loop correction to the $\bar{b}bh^0$ Yukawa interaction. Some general features that can be observed in figure 6, or extracted from the analytic expressions are:

(i) In the assumed range of SUSY parameters, the sign of ϵ_b (ϵ_t) is correlated (anti-correlated) with the sign of μ, since the gluino contribution (first term in (2.19)) dominates and the function J_3 is always negative. The term proportional to $\widetilde{A}_t$ can overcome the gluino contribution and change the sign of ϵ_b only for negative $\widetilde{A}_t$ in the parameter-space region where $|\widetilde{A}_t|/\mu^2$ is much larger than $1/M_3$. Similarly, large negative $\widetilde{A}_b$ can make ϵ_t positive, when $y_b^2|\widetilde{A}_b|/\mu^2 \gg 1/M_3$.

(ii) Since ϵ_b cannot reach large negative values, there is no significant enhancement of the bottom Yukawa coupling from the large-$\tan\beta$ resummation. The numerical effect of the terms proportional to $y_b^2\widetilde{A}_b$ is therefore small compared to those proportional to $y_t^2\widetilde{A}_t$. We verified that the scatter plots for negative $\widetilde{A}_b$ are similar to the ones with positive $\widetilde{A}_b$ shown in figure 6.

(iii) Larger values of $|\epsilon_t|$ ($|\epsilon_b|$) tend to be correlated with larger values of $|h_1|$ ($|h_2|$). However, there is no strict relation, since the dominant contribution to the "wrong-Higgs" Yukawa couplings is proportional to the gluino mass, while those to the Higgs-sector couplings involve the A terms.

(iv) The largest contribution to h_1 is given by the term proportional to $\widetilde{A}_t$ in (2.9). However, the sign of h_1 is not determined uniquely by the sign of $\widetilde{A}_t$ due to the gaugino term proportional to g_2^2, which is always positive (the function H_2 is positive) and can become comparable to the $\widetilde{A}_t$ term in some regions of the parameter space.

(v) In most of the explored parameter space the sign of h_2 is opposite to the sign of $\widetilde{A}_t$. Since the $\delta\lambda_7$ term in the definition (4.3) of h_2 is the larger of the two, we have $\mathrm{sgn}(h_2) = -\mathrm{sgn}(\delta\lambda_7)$. The relation $\mathrm{sgn}(\delta\lambda_7) = \mathrm{sgn}(\widetilde{A}_t)$ can be understood as follows: neglecting the smaller terms involving gauge couplings, eq. (2.14) for $\delta\lambda_7$ contains the terms

$$-3\mu\, y_t^4\widetilde{A}_t\left\{ J_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) + J_2(m_{\tilde{t}_R}, m_{\tilde{Q}})\right\} - 3\mu\,\widetilde{A}_t^3 y_t^4 K_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) \qquad (4.6)$$

proportional to powers of $\widetilde{A}_t$. The first term has the same sign as $\widetilde{A}_t$, since J_2 is always negative, while the second term has opposite sign, since K_2 is positive. Defining the ratio

$$r(x) = -\frac{J_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) + J_2(m_{\tilde{t}_R}, m_{\tilde{Q}})}{m_{\tilde{Q}}^2\, K_2(m_{\tilde{Q}}, m_{\tilde{t}_R})} \qquad (4.7)$$
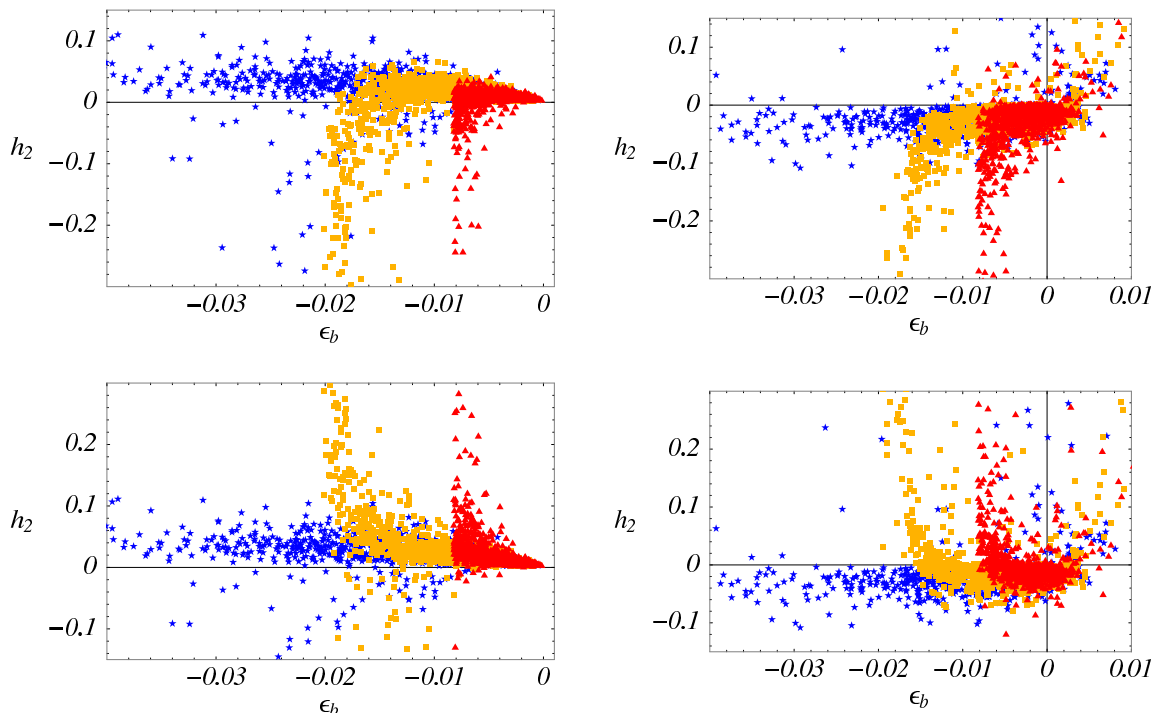
Figure 7: The correction h_2 versus ϵ_b, for $M_A = 200\,\text{GeV}$ and negative μ. The plots in the first row correspond to $\{\text{sgn}(\widetilde{A}_t), \text{sgn}(\widetilde{A}_b)\} = \{+1, +1\}$ and $\{-1, +1\}$, while those in the second row have $\{\text{sgn}(\widetilde{A}_t), \text{sgn}(\widetilde{A}_b)\} = \{+1, -1\}$, $\{-1, -1\}$. Black, lightgray and darkgray points (blue, yellow and red in colored plots) correspond to $\tan\beta = 10, 35$ and 60, respectively.

with $x = m_{\tilde{t}_R}^2 / m_{\tilde{Q}}^2$, we find that $r(x)$ is monotonically increasing, satisfying $1.6 < r(x) < 169$ when $1/100 < x < 100$ as allowed by our parameter-space sampling. The third term in (4.6) can compete with the first only if $(\widetilde{A}_t/m_{\tilde{Q}})^2 > r(x)$. For $x = 1$ this relation requires large A terms, $(\widetilde{A}_t/m_{\tilde{Q}})^2 > 6$, which are disfavoured by vacuum stability arguments. We also note that the largest gaugino term in $\delta\lambda_7$ (the one proportional to g_2^4) gives a positive contribution because the function L_2 is negative.

Let us now turn to the case $\text{sgn}(\mu) = -1$. The most important difference is that the negative sign of the μ parameter flips the sign of ϵ_b to negative values (and the one of ϵ_t to positive values), which leads to a strong increase of the bottom and τ Yukawa coupling, when ϵ_b cancels the $1/\tan\beta$ term in (3.22). The validity of a perturbative expansion is in doubt when the Yukawa couplings become too large. In the parameter space sampling for negative μ, we therefore only keep points that satisfy $0 < y_{b,\tau} < 2$.

The dependence of the Higgs sector correction h_2 versus ϵ_b, for three representative values of $\tan\beta = 10, 35, 60$, $M_A = 200\,\text{GeV}$, and all other SUSY parameters scanned randomly in the above intervals is shown figure 7. The perturbativity cut on y_b has a strong effect, since it eliminates points with large negative values ϵ_b given a value of $\tan\beta$. The value of ϵ_b at which this happens can be estimated by $\epsilon_b \tan\beta \approx -0.5$, which gives $-\epsilon_b \approx 0.05, 0.014, 0.008$ for $\tan\beta = 10, 35, 60$, respectively, in agreement with the figures.
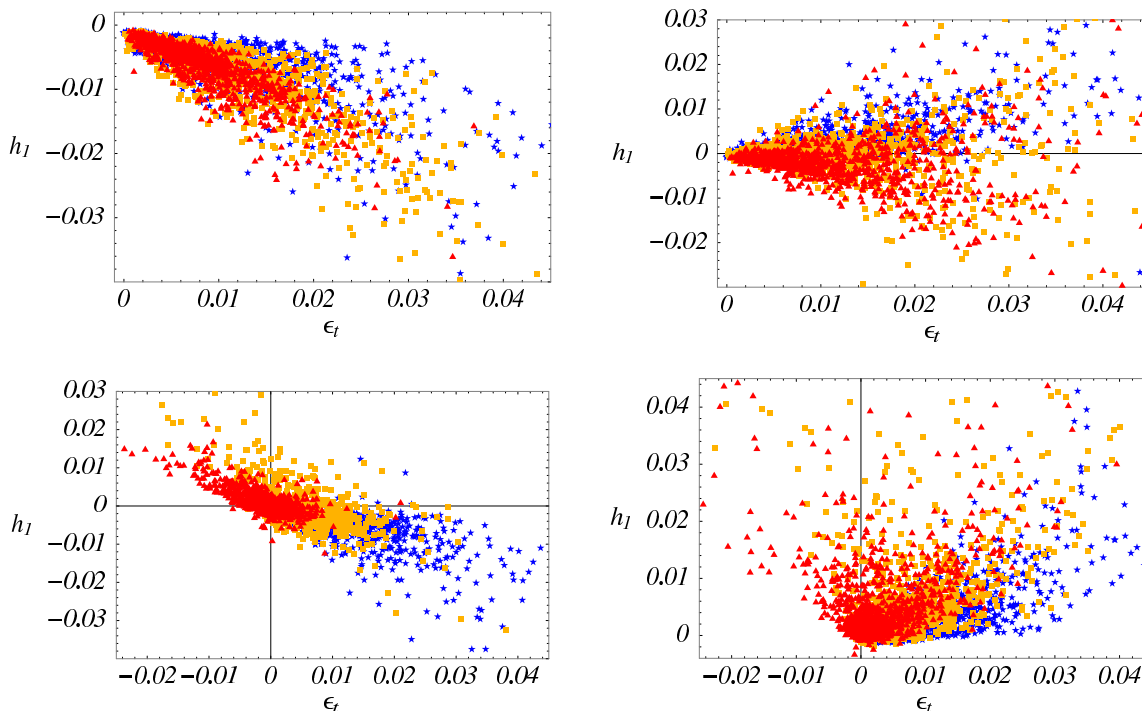
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Figure 8: The correction h_1 versus ϵ_t, for $M_A = 200\,\text{GeV}$ and negative μ. Plots in the first row have $\{\text{sgn}(\widetilde{A}_t), \text{sgn}(\widetilde{A}_b)\} = \{+1, +1\}$ and $\{-1, +1\}$, and $\{\text{sgn}(\widetilde{A}_t), \text{sgn}(\widetilde{A}_b)\} = \{+1, -1\}, \{-1, -1\}$ in the second row. Black, lightgray and darkgray points (blue, yellow and red in colored plots) correspond to $\tan\beta = 10, 35$ and 60, respectively.

At these points we observe a rapid increase of $|h_2|$ driven by the term

$$-3\mu^3\, y_b^4 \widetilde{A}_b K_2(m_{\tilde{Q}}, m_{\tilde{b}_R}) \tag{4.8}$$

in (2.14), which gives a contribution to $h_2 \sim -\delta\lambda_7$ with sign opposite to $\widetilde{A}_b$. Point (i) discussed above also applies to the case of negative μ taking into the account the reversed signs of $\epsilon_{b,t}$. Finally, figure 8 shows the Higgs correction h_1 versus ϵ_t for $\tan\beta = 10, 35, 60$. Compared to the case of positive μ, the sign of $\widetilde{A}_b$ has now a relevant effect, especially for larger values of $\tan\beta$, due to the contribution of the $\widetilde{A}_b$ term in z_{12}. However, the growth with y_b is less pronounced than in case of h_2, since h_1 does not contain terms proportional to y_b^4.

5. Summary

This paper has been motivated by previous work [3] that systematically investigated the resummation of SUSY QCD large-$\tan\beta$ effects in the decoupling limit, where the standard model particles and Higgs scalars remain light, but which did not consider electroweak effects. We performed a complete one-loop matching of the MSSM to a two-Higgs doublet model for all couplings that are absent or suppressed in the MSSM with large $\tan\beta$, keeping all contributions that are $\tan\beta$-enhanced relative to the suppressed tree terms. This

includes complete expressions for the kinetic mixing and Higgs self couplings $\delta\lambda_{5-7}$, which have not been given previously.

Our result confirms, as expected, that a resummation of large-$\tan\beta$ effects to arbitrary loop order is necessary only for the bottom and tau Yukawa couplings, if the bottom and tau mass are used as parameters of the MSSM. It clarifies a point left open in [3], namely how the renormalization of $\tan\beta$ affects this resummation. We find that the standard expression for the resummed Yukawa coupling is valid in renormalization schemes where the counterterms (shifts) of $\tan\beta$ and the vacuum expectation value v_d do not receive large-$\tan\beta$ contributions. In other schemes, a finite counterterm must be explicitly included in the resummation formula.

Besides the known "wrong-Higgs" Yukawa couplings, all other Higgs-sector effects that feed into the Yukawa couplings by modifying the definition of the physical Higgs fields at one loop can be parameterized in terms of two couplings h_1, h_2 dominated by kinetic mixing and the Higgs self-coupling $\delta\lambda_7$, respectively. The same h_2 also enters the effective trilinear Higgs couplings. Our numerical study suggests that, where present, the Higgs-sector effect h_2 is more important than the "wrong-Higgs" Yukawa couplings, and we identified the dominant SUSY parameter dependences. Our result extends and corrects a previous result [7] that included the kinetic-mixing effect, but not the one from the one-loop induced Higgs couplings.

Although obtained in the decoupling limit, the effective one-loop couplings derived in this paper should be useful to obtain simple analytic estimates of the leading quantum corrections to those Yukawa and Higgs trilinear interactions, that are suppressed at tree-level, such as the $\bar{b}bh^0$ coupling.

Acknowledgments

References

[1] A. Dabelstein, *Fermionic decays of neutral MSSM Higgs bosons at the one loop level*, *Nucl. Phys.* **B 456** (1995) 25 [`hep-ph/9503443`].

[2] J.A. Coarasa, R.A. Jimenez and J. Solà, *Strong effects on the hadronic widths of the neutral Higgs bosons in the MSSM*, *Phys. Lett.* **B 389** (1996) 312 [`hep-ph/9511402`].

[3] M.S. Carena, D. Garcia, U. Nierste and C.E.M. Wagner, *Effective Lagrangian for the $\bar{t}bH^+$ interaction in the MSSM and charged Higgs phenomenology*, *Nucl. Phys.* **B 577** (2000) 88 [`hep-ph/9912516`].

[4] L.J. Hall, R. Rattazzi and U. Sarid, *The top quark mass in supersymmetric* SO(10) *unification*, *Phys. Rev.* **D 50** (1994) 7048 [`hep-ph/9306309`].

[5] A. Freitas and D. Stöckinger, *Gauge dependence and renormalization of* $\tan\beta$ *in the MSSM*, *Phys. Rev.* **D 66** (2002) 095014 [hep-ph/0205281].

[6] N. Baro, F. Boudjema and A. Semenov, *Automatised full one-loop renormalisation of the MSSM I: the Higgs sector, the issue of* $\tan\beta$ *and gauge invariance*, *Phys. Rev.* **D 78** (2008) 115003 [arXiv:0807.4668].

[7] A. Freitas, E. Gasser and U. Haisch, *Supersymmetric large* $\tan\beta$ *corrections to* $\Delta M_{d,s}$ *and* $B_{d,s} \to \mu^+\mu^-$ *revisited*, *Phys. Rev.* **D 76** (2007) 014016 [hep-ph/0702267].

[8] S. Trine, *The MSSM Higgs sector and* B-$\bar{B}$ *mixing for large* $\tan\beta$, arXiv:0710.4955.

[9] J. Rosiek, *Complete set of Feynman rules for the minimal supersymmetric extension of the standard model*, *Phys. Rev.* **D 41** (1990) 3464 [*Erratum* hep-ph/9511250].

[10] J.A. Aguilar-Saavedra et al., *Supersymmetry parameter analysis: SPA convention and project*, *Eur. Phys. J.* **C 46** (2006) 43 [hep-ph/0511344].

[11] A. Dabelstein, *The one loop renormalization of the MSSM Higgs sector and its application to the neutral scalar Higgs masses*, *Z. Physik* **C 67** (1995) 495 [hep-ph/9409375].

[12] P.H. Chankowski, S. Pokorski and J. Rosiek, *Complete on-shell renormalization scheme for the minimal supersymmetric Higgs sector*, *Nucl. Phys.* **B 423** (1994) 437 [hep-ph/9303309].

[13] R. Hempfling and B.A. Kniehl, *On the relation between the fermion pole mass and MS Yukawa coupling in the standard model*, *Phys. Rev.* **D 51** (1995) 1386 [hep-ph/9408313].

[14] B.A. Kniehl, *Radiative corrections for* $H \to f\bar{f}(\gamma)$ *in the standard model*, *Nucl. Phys.* **B 376** (1992) 3.

[15] S. Marchetti, S. Mertens, U. Nierste and D. Stöckinger, $\tan\beta$-*enhanced supersymmetric corrections to the anomalous magnetic moment of the muon*, arXiv:0808.1530.

[16] M.S. Carena, D. Garcia, U. Nierste and C.E.M. Wagner, $b \to s\gamma$ *and supersymmetry with large* $\tan\beta$, *Phys. Lett.* **B 499** (2001) 141 [hep-ph/0010003].

[17] M. Ciuchini, G. Degrassi, P. Gambino and G.F. Giudice, *Next-to-leading QCD corrections to* $B \to X_s\gamma$ *in supersymmetry*, *Nucl. Phys.* **B 534** (1998) 3 [hep-ph/9806308].

[18] A. Djouadi, W. Kilian, M. Muhlleitner and P.M. Zerwas, *Testing Higgs self-couplings at* e^+e^- *linear colliders*, *Eur. Phys. J.* **C 10** (1999) 27 [hep-ph/9903229].

[19] Z.-Z. Xing, H. Zhang and S. Zhou, *Updated values of running quark and lepton masses*, *Phys. Rev.* **D 77** (2008) 113016 [arXiv:0712.1419].

[20] H.E. Haber and R. Hempfling, *The renormalization group improved Higgs sector of the minimal supersymmetric model*, *Phys. Rev.* **D 48** (1993) 4280 [hep-ph/9307201].

[21] J.A.M. Vermaseren, *Axodraw*, *Comput. Phys. Commun.* **83** (1994) 45.

[22] D. Binosi and L. Theussl, *JaxoDraw: a graphical user interface for drawing Feynman diagrams*, *Comput. Phys. Commun.* **161** (2004) 76 [hep-ph/0309015].